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A Modified Ant Colony Optimization Algorithm for Solving a Transportation Problem

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Authors' contributions

This work was carried out in collaboration among all authors. Author EMUSBE designed the study, managed the data collection and literature searches, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Author WBD was the first supervisor for the work. Authors SPCP and ZAMSJ were co-supervisors of the work. All authors read and approved the final manuscript.

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Abstract

Transportation of products from sources to destinations with minimal total cost plays a key role in logistics and supply chain management. The transportation problem (TP) is an extraordinary sort of Linear Programming problem where the objective is to minimize the total cost of disseminating resources from several various sources to several destinations. Initial feasible solution (IFS) acts as a foundation of an optimal cost solution technique to any TP. Better is the IFS lesser is the number of iterations to reach the final optimal solution. This paper presents a meta-heuristic algorithm, modified ant colony optimization algorithm (MACOA) to attain an IFS to a Transportation Problem. The proposed algorithm is straightforward, simple to execute, and gives us closeness optimal solutions in a finite number of iterations. The efficiency of this algorithm is likewise been advocated by solving validity and applicability examples An extensive numerical study is carried out to see the potential significance of our modified ant colony optimization algorithm (MACOA). The comparative assessment shows that both the MACOA and the existing JHM are efficient as compared to the studied approaches of this paper in terms

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of the quality of the solution. However, in practice, when researchers and practitioners deal with large-sized transportation problems, we urge them to use our proposed MACOA due to the time-consuming computation of JHM. Therefore this finding is important in saving time and resources for minimization of transportation costs and optimizing transportation processes which could help significantly to improve the organization's position in the market.

Keywords: Modified ant colony algorithm; transportation problem; initial feasible solution; Vogel's approximation method; Juman and Hoque's method.

1 Introduction

Dorigo and Maniezzo [1] developed the first foraging ant's algorithm which is called Ant Colony Algorithm (ACA). This is based on the probabilistic technique for solving various types of optimization problems. Such as vehicle routing problem [2,3,4], traveling salesman problem, traffic assignment problem [5], shortest path problem, minimum spanning tree problem, etc. The algorithm is examining for an optimal path in the graph based on the behavior of ants.

On the other hand, the development of this algorithm was ants can find the shortest roots between food sources and their colony. Ants are social insects and walk randomly. Ants communicate to each other by setting down pheromones along their path, so where ants go inside and around their subterranean insect state is a stigmergic framework. In numerous ant species, ants strolling from or to a food source, deposit on the ground a substance called pheromone.. Different ants can smell this pheromone, and its essence impacts the decision of their way, that is, they will in general follow solid pheromone fixations. The pheromone deposited on the ground shapes a pheromone trail, which permits the ants to discover great of food that have been recently recognized by different ants. Utilizing arbitrary strolls and pheromones inside a ground containing one nest and one food source, the ants will leave the nest, discover the food and return to the nest.After some time, the way being utilized by the ants will converge to the shortest path.

Generally, ACA is very easy to understand and its applications provide very good results. Very recently, Chowdhury et al. (Article-in-press) [6] proposed a modified ACA to solve a real-life dynamic traveling salesman problem.

The transportation problem (TP) deals with the distribution of a product manufactured at factories to some various warehouses. The objective of this problem is to determine the feasible amounts to be shipped from each source to each destination with a minimum total transportation cost.

The TP was first developed by Hitchcock [7] and then Koopmans [8] developed Optimum Utilization of Transportation System. After that, Charnes and Cooper [9] developed the stepping stone method and Dantzig [10, 11] developed the transportation Simplex Method to this problem.

Besides, several heuristic solutions approach such as Northwest Corner Method [12], minimum cost method [12], VAM -Vogel's approximation method [13,14,15,16], JHM –Juman and Hoque's method [17], GVAM –Goyal's version of VAM [18], EHA-An Efficient Heuristic Approach [19], etc were proposed to obtain an Initial Feasible Solution (IFS) to the TP. Moreover, Sabbagh [20] developed a new hybrid algorithm for balanced TP. Sharma [21] presented a new solution procedure to solve the dual of the incapacitated transportation problem, Juman and Nawarathne [22] presented an alternative approach to solving a TP.

This paper presents an overview of the concept of ACA and provides a review of its applications for solving both types of balance and unbalance TP. The proposed one is very simple, easy to understand and it always tries to reach a minimum cost solution to the concern problem. In this paper, several modifications to this ACA are made and ensured a solution that is very closer to the optimal solution with less iteration.

The remainder of this paper is as follows: Section 2 deals with the mathematical formulation of the transportation problem. In Section 3 the modified ACA is proposed and illustrated with a numerical example problem. Then, its comparative studies with the existing ones on the results of some benchmark instances are carried out to show the potential significance of the proposed approach. Finally, conclusions along with limitations and future research directions are presented in Section 5.

2 Mathematical Formulation

The TP is focused on the distribution of the product from $\boldsymbol{m}$ sources having capacities$(a_1, a_2, \ldots, a_m)$ to $\boldsymbol{n}$ destinations having demands$(b_1, b_2, \ldots, b_n)$ to meet the demand of each destination with the least total transportation cost. Arc (i, j) joining source i to destination j carries two pieces of information: the transportation cost per unit,c_{ij} , and the amount shipped, X_{ij}, where $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

Hence the above problem (sometimes called as the general, classical or Hitchcock transportation problem) can be given in the following form:

$$\text{Min} \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \ , \quad i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \ , \quad j = 1, 2, \ldots, n$$

$$\text{Where} \quad x_{ij} \geq 0 \ \forall\ i,\ j$$

A transportation problem is said to be balanced if the total supply from all sources is equal to the total demands

at all destinations. That is, $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ otherwise, it is called an unbalanced transportation problem.

3 Methodology

3.1 Solution procedure

Here, we propose a meta-heuristic solution method based on Ant Colony Optimization Algorithm (ACOA) to solve the transportation problem. This strategy animated by the behavior of veritable ant colonies and it has been successfully applied to solve several combinatorial optimization problems and has achieved agreeable performances [23,24,25,26]. Following Afshar [27] the basic steps on this algorithm [1,28] can be defined as follows:

Step 1 ***m ants*** are randomly placed on the *n* decision points and the amount of pheromone trail on all arcs is initialized to some proper value at the start of the computation.

Step 2: A transition rule is used at each decision point i to decide which option is to be selected. Once the option at the current decision point is selected, the ants move to the next decision point and the solutions are incrementally created by ants as they move from one point to the next one. This procedure is repeated until all decision points of the problem are covered. The transition rule used in the original ant system is defined as follows (Dorigo et al. [1]):

The ants are driven by a probability rule to pick their answer for the problem, known as a visit. The probability rule between two i hubs j, called Transition Rule [29,30] and it be dependent on upon two factors: the heuristic and metaheuristic. The transition rule evaluates the probability of ant k, situated at city i, going to city j and it is given by:

$$P_{ij} = \frac{{\varphi_{ij}}^{\alpha} {\omega_{ij}}^{\beta}}{\sum_{j=}^{m} {\varphi_{ij}}^{\alpha} {\omega_{ij}}^{\beta}}$$

This equation calculates the probability of selecting a single component of the solution. In this research, $\omega_{ij} = \frac{1}{c_{ij}+\theta}$: $\alpha > 0$, $\beta > 0$ .Where θ is linear and $\theta =$ min.valuec_{ij}; cost between node i and node j, $\theta \neq 0$.

For our scenario, we assumed $\alpha = \beta = 1$ in the transition rule,

$$P_{ij} = \frac{\varphi_{ij}\left[\frac{1}{c_{ij}+\theta}\right]}{\sum_{j=}^{m} \varphi_{ij}\left[\frac{1}{c_{ij}+\theta}\right]}; \ i^{\text{th}} \text{ ant visits the } j^{\text{th}} city$$

$$0 \qquad ; \text{ Otherwise}$$

Pheromone update rules are used to calculate the amount of pheromone level on each edge between node i and node j. The solution of each ant is updated using following update function:

$$\varphi_{ij}(t+1) = (1-\rho)\varphi_{ij}(t) + \sum_{k=1}^{m} \Delta{\varphi^{k}}_{ij}(t)$$

Where,

$\Delta{\varphi^{k}}_{ij} = \frac{\aleph}{L^k}$; if component $(i, j)$was used by ant (best route)

= 0 ; Otherwise

Here, $L^k$is the distance of the best route. $\aleph$ Is simply a parameter to adjust the amount of pheromone deposited, typically it would be set to 1. We sum $\frac{\aleph}{L^k}$for every solution which used component (i, j), then that value becomes the amount of pheromone to be deposited on component (i, j).

Where $\varphi_{ij}(t) = \frac{1}{min\ (no.demand, no.supply)}$,ρ represents the pheromone evaporation rate and $0 < \rho \leq 1$.

3.2 Modified Ant Colony Optimization Algorithm (MACOA)

The steps of the proposed method can be expressed as follows:

Step 1: If the TP is unbalanced, convert it to a balanced TP by adding a dummy row or a dummy column accordingly.

Step 2: Compute the probability using the Transition Rule and form the probability matrix of order m x n.

Step 3: Ants in the colonies are placed at the starting nodes with the maximum value of the probability in the probability matrix to make the first allocation.

Step 4: Identify the min (a_i, b_j) in the demand or supply cell. Allocate the particular min (a_i, b_j) to cell corresponding to the maximum probability.

Step 5: If the demand in the column (or supply in the row) is satisfied, remove that column (or row) and move to the next maximum probability cell.

Step 6: If the termination condition is satisfied ($i.e. a_i = b_j = 0$), then go to Step 8. Otherwise go to Step 4.

Step 7: Stop and determine solution.

The MACOA is validated using a numerical example. Detailed computation of this example using the proposed method (MACOA) is provided in Appendix F. The flow chart representation of the above MACOA is also provided below:

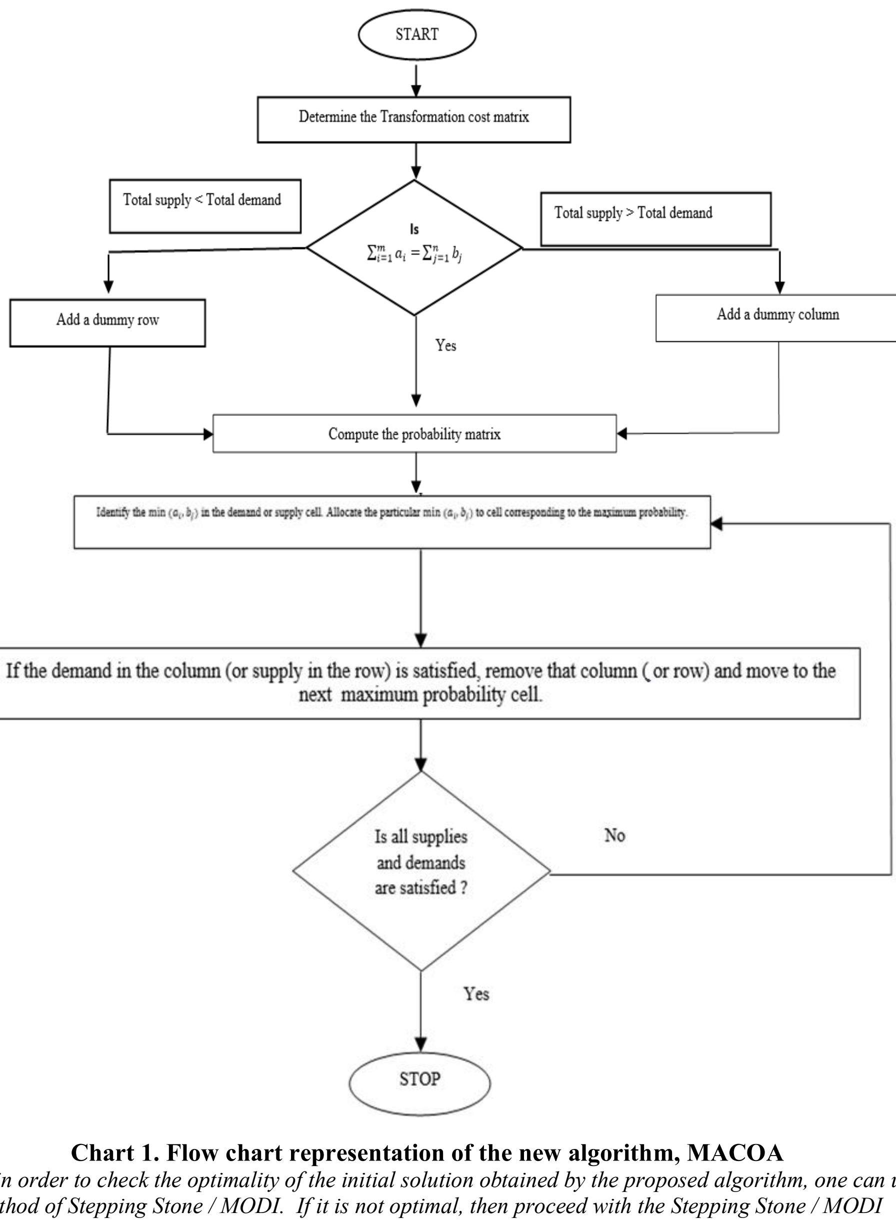

Chart 1. Flow chart representation of the new algorithm, MACOA

Note that, in order to check the optimality of the initial solution obtained by the proposed algorithm, one can use the method of Stepping Stone / MODI. If it is not optimal, then proceed with the Stepping Stone / MODI

4 Comparative Assessment

This section provides performance comparisons across the various well-known methods – LCM, NWCM, ZSM, JHM, VAM, and some other popular methods by the solutions obtained from disparate problems. Comparative assessments are performed and illustrated in the immediately following sections. The detailed representation of the numerical data of Table 1 is provided in Appendix A [31].

Table 1. Comparative results of LCM, VAM, and New method for 12 benchmark instances

Problem chosen from	TCIFS(I_{FS})			Optimal cost(O_c)	Percentage minimal	Deviation total	From cost
	LCM	VAM	NEW		LCM	VAM	NEW
Deshmukh [32]	555	555	555	555	0.00	0.00	0.00
Deshmukh [32]	114	112	112	112	1.78	0.00	0.00
Ahmed (2016)	2,900	2,850	2,850	2,850	1.75	0.00	0.00
Ahmed (2016)	3,500	3,320	3,320	3,320	5.42	0.00	0.00
Korukoglu [13]	72,174	59,356	59,356	59,356	21.59	0.00	0.00
Imam et al. [33]	475	475	475	475	0.00	0.00	0.00
Taylor (Module B)	4,550	4,525	4,525	4,525	0.55	0.00	0.00
Deshmukh [32]	305	290	260	260	17,3	11.53	0.00
Sen et al. [34]	2,404,500	2,164,000	2,146,750	2,146,750	11.20	0.80	0.00
Ahmed (2016)	9,800	9,200	9,200	9,200	6.52	0.00	0.00
Ahmed (2016)	14,625	13,225	10,375	10,375	40.96	27.46	0.00
Ahmed (2016)	6,450	6,000	5,600	5,600	15.17	7.14	0.00

The comparative results obtained in Table 1 are also depicted using bar graphs and the results are given in Fig. 1.

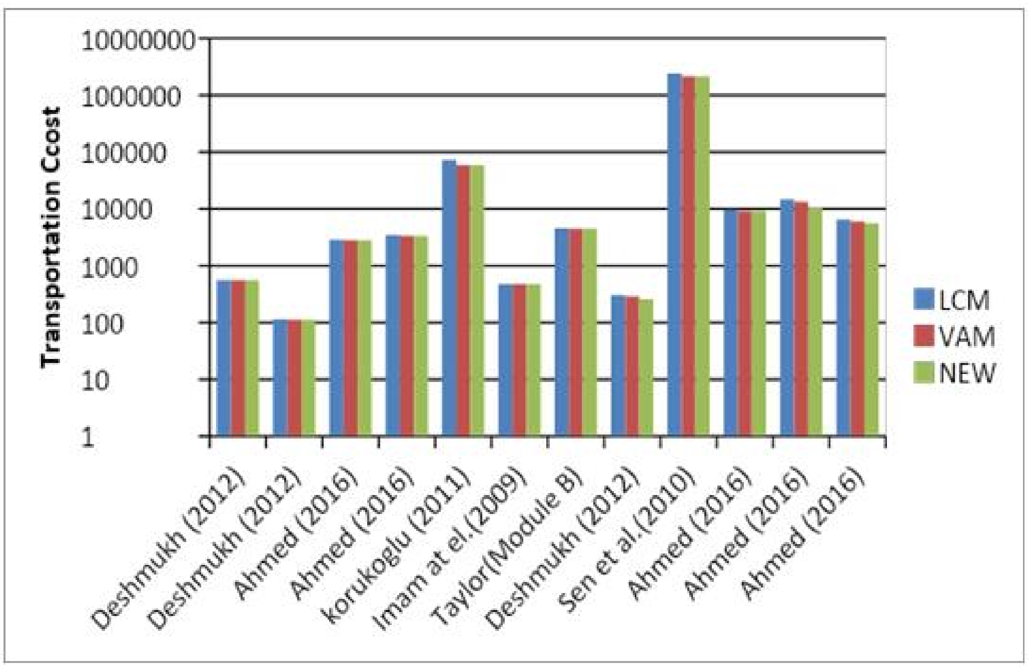

Fig. 1. Comparative study of the result obtained by LCM, VAM with the proposed method

Line graphs for the percentage deviation (of the LCM, VAM with the proposed method from minimal total cost solution) obtained in Table 1 are presented in Fig.2.

It can be seen from Table 1, and Fig. 1 and Fig. 2 that the proposed method (MACA) is more efficient than LCM and VAM in every case where an improvement in efficiency was possible. Thus, proposed method of this paper performs better compared to LCM and VAM.

Performance measure of NEM over ZSM, VAM and JHM for 9 benchmark instances is shown in the Table 2. The detailed representation of the numerical data of Table 2 is provided in Appendix B.

The comparative results obtained in Table 2 are also depicted using bar graphs and the results are given in Fig. 3.

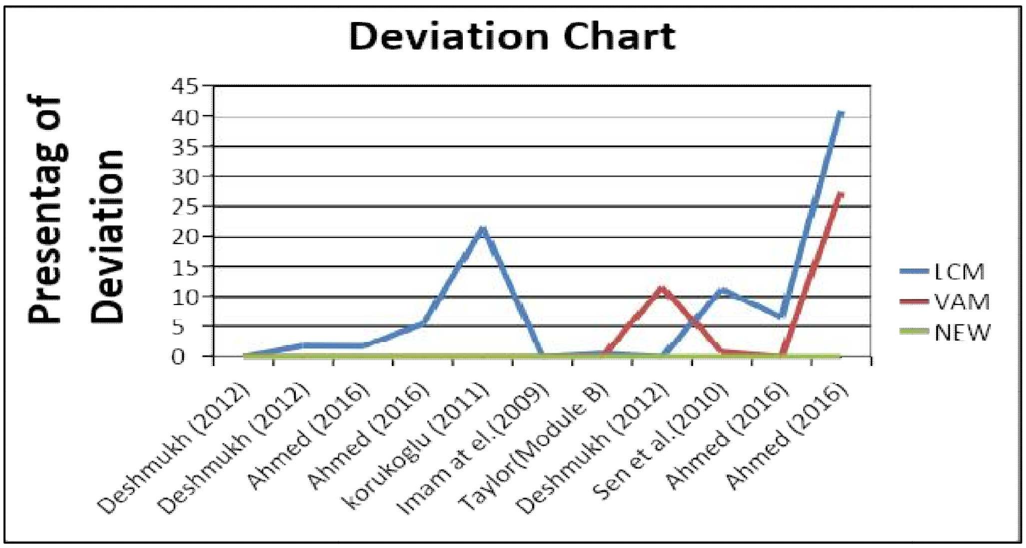

Fig. 2. Percentage of deviation of the results obtained by LCM, VAM and the proposed method

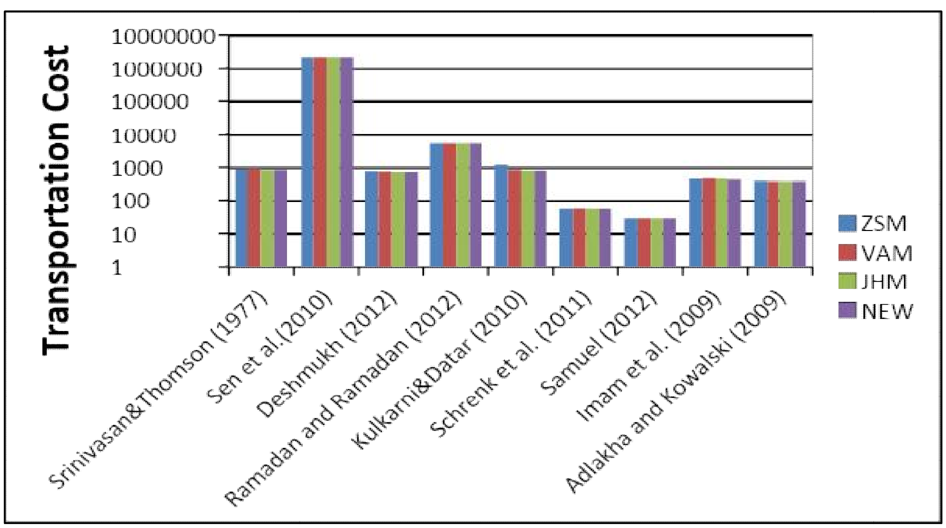

Fig. 3. Comparative study of the result obtained by ZSM, VAM, JHM with the proposed method

Table 2. A comparative study of ZSM, VAM, JHM and NEM for 9 benchmark instances

Problem chosen from Juman and Hoque [17]	TCIFS (I_{FS})				Optimal cost(O_c)	Percentage minimal	Deviation total		From cost
	ZSM	VAM	JHM	NEW		ZSM	VAM	JHM	NEW
Srinivasan & Thomson [35]	910	955	880	880	880	3.40	8.52	0.00	0.00
Sen et al. [34]	2158500	2164000	2146750	2146750	2146750	0.55	0.80	0.00	0.00
Deshmukh [32]	798	779	743	743	743	7.40	4.84	0.00	0.00
Ramadan and Ramadan [36]	5,600	5600	5600	5600	5600	0.00	0.00	0.00	0.00
Kulkarni & Datar [37]	1,200	880	840	840	840	42.85	4.76	0.00	0.00
Schrenk et al. [38]	59	60	59	59	59	0.00	1.69	0.00	0.00
Samuel [39]	28	28	28	28	28	0.00	0.00	0.00	0.00
Imam et al. [33]	460	475	460	435	435	0.06	3.26	0.00	0.00
Adlakha and Kowalski [40]	400	390	390	390	390	2.56	0.00	0.00	0.00

Table 3. A comparative results obtained by ZSM, VAM, JHM and New method for the seven benchmark instances

Problem chosen from Juman and Hoque [17]	TCIFS(I_{FS})				Optimal cost(O_c)	Percentage minimal	Deviation total		From cost
	ZSM	VAM	JHM	NEW		ZSM	VAM	JHM	NEW
Problem 1.	4525	5125	4525	4525	4525	3.40	8.52	0.00	0.00
Problem 2.	3460	3520	3460	3460	3460	0.55	0.80	0.00	0.00
Problem 3.	920	960	920	920	920	7.40	4.84	0.00	0.00
Problem 4.	864	859	809	809	809	0.00	0.00	0.00	0.00
Problem 5.	475	475	417	417	417	42.85	4.76	0.00	0.00
Problem 6.	3598	3778	3458	3458	3458	4.04	9.25	0.00	0.00
Problem 7.	136	112	109	109	109	0.00	0.00	0.00	0.00

Line graphs for the percentage deviation (of the ZSM, VAM, JHM and the proposed method) from minimal total cost solution obtained in Table 2 are presented in Fig. 4.

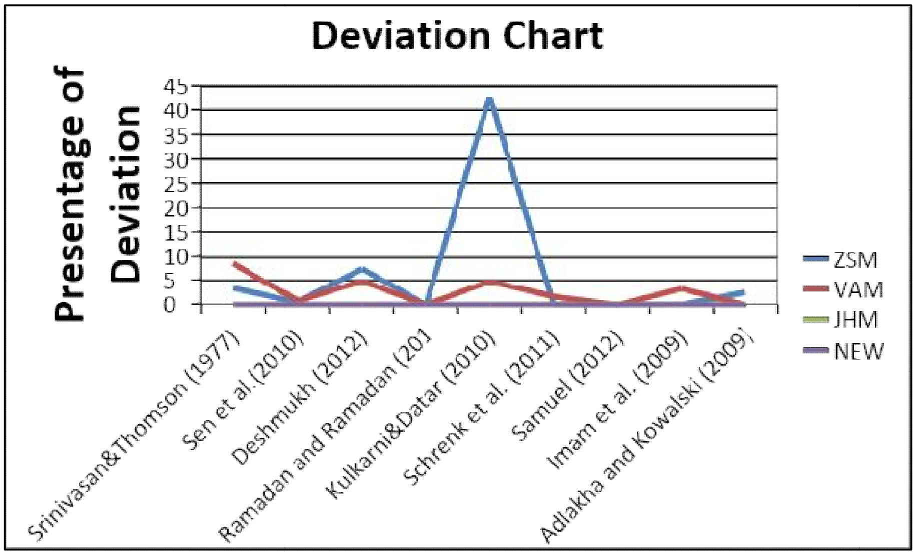

Fig. 4. Percentage of deviation of the results obtained by ZSM, VAM, JHM and the proposed method

According to the simulation results (Table 2, Fig. 3 and Fig. 4),the proposed method yields better results to all problems in Table 2 compared with ZSM and VAM. It provides the same results as JHM.

Comparative results obtained by ZSM, VAM, JHM ,and the proposed method for the seven benchmark instances are shown in the following Table 3. Detailed data representation of these seven problems is provided in Appendix C.

The comparative results obtained in Table 3 are also depicted using bar graphs and the results are given in Fig. 5.

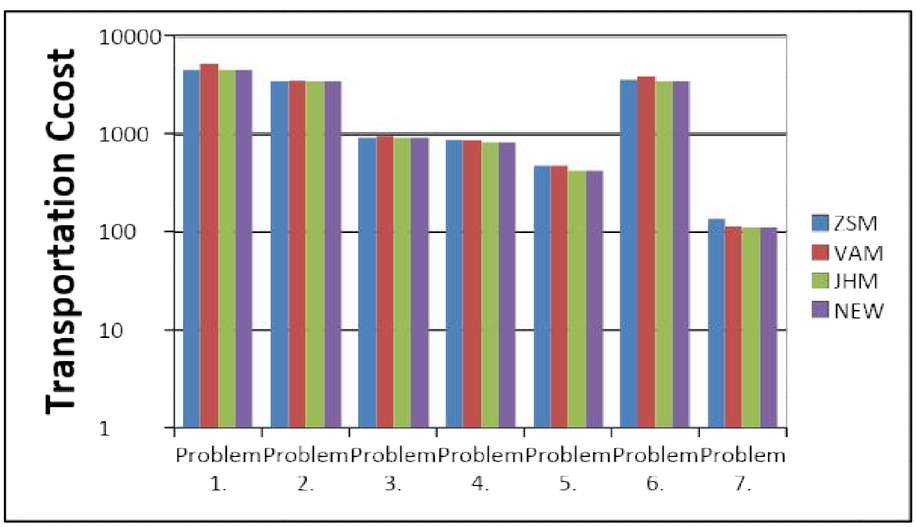

Fig. 5. Comparative study of the result obtained by ZSM, VAM, JHM with NEW method

Line graphs for the percentage deviation (of the ZSM, VAM, JHM and New method) from minimal total cost solution obtained in Table 3 are depicted in Fig. 6.

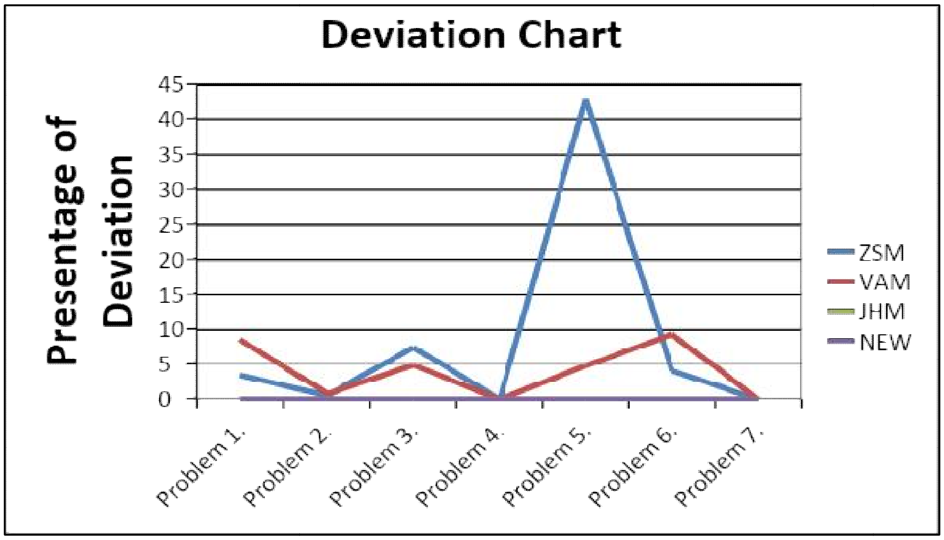

Fig. 6. Percentage of deviation of the results obtained by ZSM, VAM, JHM and the proposed method

Based on the above results (Table 3, Fig. 5 Fig. 6), the proposed method outperforms ZSM and VAM. It provides the same results as JHM. However, this cannot deny the value of our proposed method, modified ant colony algorithm. Note that, here, the formula is $D = \frac{I_{FS-}O_C}{O_c} \times 100$ used to obtain the percentage deviation from the optimal result. This calculation is carried out to evaluate how much nearer the I_{FS} is to O_c.

In addition to the above benchmark instances, we have also studied some other numerical example problems chosen from Ahmed [41] in order to determine the performance of our new method over the available 14 approaches. The obtained results are presented in Table4. Detailed data representation of these four example problems is provided in Appendix D.

Table 4. Comparative results of new method along with the 14 available approaches for 4 benchmark instances

Methods.	**Total initial**	**cost feasible**	**for solution**	**the**
	Ex.1	**Ex.2**	**Ex.3**	**Ex.4**
North West Corner Method(NWCM)	4400	4,160	540	1,500
Row Minimum Method (RMM)	2,850	4,120	470	1,450
Column Minimum Method (CMM)	3,600	3,320	435	1,500
Least Cost Method (LCM)	2,900	3,500	435	1,450
Vogel's Approximation Method (VAM)	2,850	3,320	470	1,500
Extremum Difference Method (EDM)	2,900	3,620	415	1,390
Highest Cost Difference Method(HCDM)	2,900	3,620	435	1,450
Average Cost Method (ACM)	2,900	3,320	455	1,440
TOCM-MMM Approach	2,900	3,620	435	1,450
TOCM-VAM Approach	2,850	3,620	430	1,450
TOCM-EDM Approach	2,850	3,620	435	1,450
TOCM-HCDM Approach	2,900	3,620	435	1,450
TOCM-SUM Approach	2,850	3,320	455	1,440
ATM Approach ATM	2,850	3,320	415	1,390
Proposed New Method	**2,850**	**3,320**	**410**	**1,390**
Optimal Solution	2,850	3,320	410	1,390

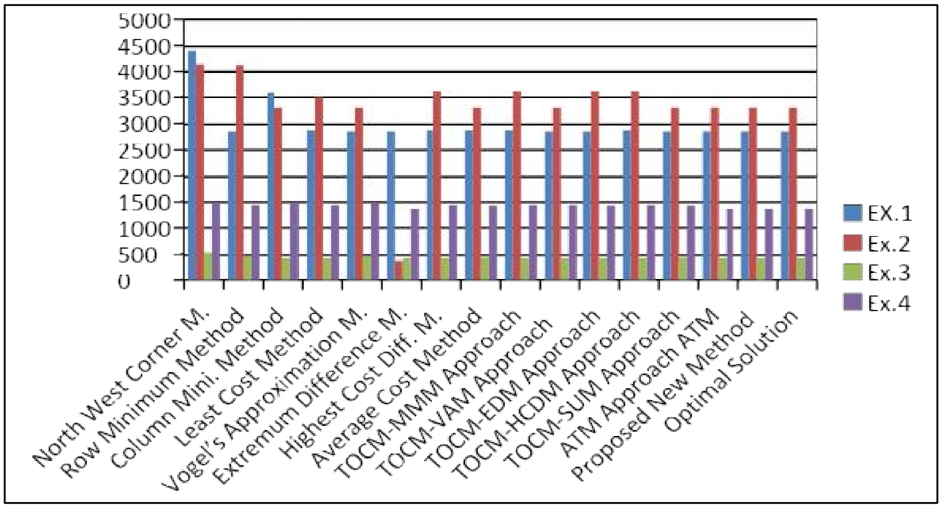

Fig. 7. Comparative study of the result obtained by the proposed method against the existing 14 approaches

Note that, although our proposed method yields the optimal solution to the above four benchmark instances, the available fourteen approaches do not. We have also studied a further set of benchmarks in determining the performance measure of the proposed method (PM) over NWCM, LCM, VAM, and EDM. Detailed data representation of the problems is given in Appendix E [42]

Table 5. Performance measure of new method (NM) over NWCM, LCM, VAM and EDM

Solution methods	Total cost for the initial feasible solution	
	Ex.1	**Ex.2**
NWCM	107	19,700
LCM	83	13,100
VAM	80	12,250
EDM	83	12,250
PM	**76**	**11,500**
Optimal Solution	76	11,500

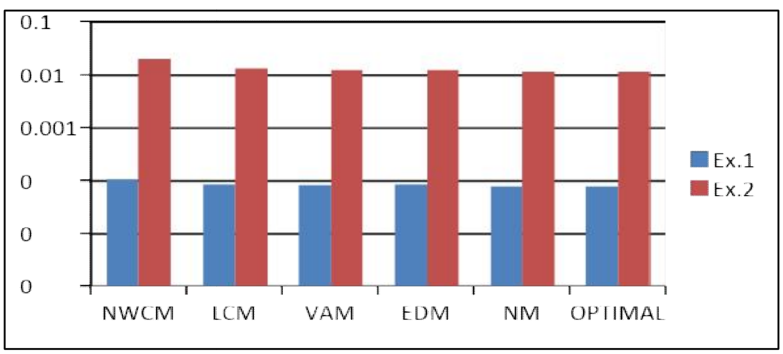

Fig. 8. Comparative study of the result obtained by PMagainst the existing NWCM, LCM, VAM, EDM

Note that, although PM yields the optimal solution to both benchmark instances above, the available four approaches (NWCM, LCM, VAM, and EDM) do not. Hence, the comparative assessments of the above different cases show that both the modified ant colony algorithm and JHM are efficient as compared to the studied approaches of this paper in terms of the accuracy of the solution

The comparative results obtained in Table 5 are also depicted using bar graphs and the results are given in Fig. 8.

5 Conclusion

The transportation problem is one of the significant problems in the field of operation research for its wide application, in reality. It is related to finding the minimum cost transportation plan for moving various origins to different destinations. Transportation problem (TP) is one of the special class of Linear Programming problems in which the objective is transportation problem solution methods to reach the optimal solution. Many researchers have paid attention to solve this problem using different approaches. Out of all the existing methods in the literature, Northwest, Least Cost, Vogel's Approximation, and Juman and Hoque's methods are the most prominent and renowned methods in finding an initial feasible solution to a TP. Modified Distribution (MODI) Method and Stepping Stone Method are the most acceptable methods in finding the minimal total cost solution to the transportation problem. These well-known minimal total cost solution techniques start with an Initial Feasible Solution (IFS). Thus an IFS acts as a foundation of an optimal cost solution technique to any TP. Better is the IFS lesser is the number of iterations to reach the final optimal solution. However, in this research paper, we discuss a new alternative method, a modified ant colony optimization algorithm which gives often an optimal solution to the transportation problem.

In this research paper, we first examine different initial solutions providing methods for attaining initial feasible solutions to balanced and unbalanced transportation problems. This research paper presents an overview of the concept of an Ant colony algorithm and provides a review of its applications to solve transportation problems. The PM is very simple, easy to understand, and easy to implement. Several modifications to the ant colony algorithm are made and ensured a solution which is very closer to the optimal solution. This method requires a minimum number of steps to reach the optimality as compared to the existing methods. An extensive numerical study is carried out to see the potential significance of our modified ant colony algorithm (MACOA). The comparative assessment shows that both the MACOA and JHM are efficient as compared to the studied approaches of this paper in terms of the quality of the solution. Also, A comparative study shows that the new method gives the minimal total cost solution to 34 out of 34 benchmark instances. However, in practice, when researchers and practitioners deal with large-sized transportation problems, we urge them to use our proposed MACOA due to the time-consuming computation of JHM.

Although our proposed Modified Ant Colony Optimization algorithm provides better IFS that is often optimal, it does not always guarantee the exact optimal cost solution. Since the existing well-known exact optimal cost solution technique (SSM) deals fully with the path tracing technique, it becomes very difficult in solving large-scaled transportation problems. Thus, we intend to devote ourselves in near future in proposing an alternate exact optimal approach that gets rid of this difficulty.

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Competing Interests

Authors have declared that no competing interests exist.

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Appendix A

Problem chosen from.	Size	Data of the problem
1.BTP-1 (12)	3 × 3	$c_{ij} = [6,4,1; 3,8,7; 4,42], s_i = [50,40,60], d_j = [20,95,35]$
2.BTP-2 (12)	4 × 6	$c_{ij} = [9,12,9,6,9,10; 7,3,7,7,5,5; 6,5,9,11,3,11; 6,8,11,2,2,10], s_i = [5,6,2,9], d_j = [4,4,6,2,4,2]$
3.BTP-3(4)	3 × 4	$c_{ij} = [3,1,7,4; 2,6,5,9; 8,3,3,2], s_i = [300,400,500], d_j = [250,350,400,200]$
4.BTP-4(4)	3 × 4	$c_{ij} = [50,60,100,50; 80,40,70,50; 90,70,30,50], s_i = [20,38,16], d_j = [10,18,22,24]$
5.BTP-(24)	5 × 5	$c_{ij} = [46,74,9,28,99; 12,75,6,36,48; 35; 199,4,5,71; 61,81,44,88,9; 85,60,14,25,79], s_i = [461,277,356,488,393], d_j = [278,60,461,116,1060]$
6.BTP-(18)	3 × 4	$c_{ij} = [10,2,20,11; 12,7,9,20; 4,14,16,18], s_i = [15,25,10], d_j = [5,15,15,15]$
7.BTP-7 (21)	3 × 3	$c_{ij} = [6,8,10; 7,11,11; 4,5,12], s_i = [150,175,275], d_j = [200,100,300]$
8.UTP-1 (12)	3 × 5	$c_{ij} = [4,1,2,4,4; 2,3,2,2,3; 3,5,2,4,4], s_i = [60,35,40], d_j = [22,45,20,18,30]$
9.UTP-2 (35)	5 × 4	$c_{ij} = [60,120,75,180; 58,100,60,165; 62,110,65,170; 65,115,80,175; 70,135,85,195], s_i = [8000,9200,6250,4900,6100], d_j = [5000,2000,10000,6000]$
10.UTP-3 (5)	3 × 5	$c_{ij} = [5,8,6,6,3; 4,7,7,6,5; 8,4,6,6,4], s_i = [800,500,900], d_j = [400,400,500,400,800]$
11. UTP-4 (5)		$c_{ij} = [12,10,6,13; 19,8,16,25; 17,15,15,20; 23,22,26,12], s_i = [150,500,600,225], d_j = [300,500,75,100]$
12.UTP-5(5)	4 × 4	$c_{ij} = [5,4,8,6,5; 4,5,4,3,2; 3,6,5,8,4], s_i = [600,400,1000], d_j = [450,400,200,250,300]$

Appendix B

Problem chosen from Juman&Hoque [23]	Data of the problem
Problem 1 [39]	$c_{ij} = [3,6,3,4; 6,5,11,15; 1,3,10,5], s_i = [80,90,55], d_j = [70,6035,60]$
Problem 2 [36]	$c_{ij} = [60,120,75,180; 58,100,60,165; 62,110,65,170; 65,115,80,175; 70,135,85,195], s_i = [8000,9200,6250,4900,6100], d_j = [5000,2000,10000,6000]$
Problem 3 [12]	$c_{ij} = [19,30,50,10; 70,30,40,60; 40,8,70,20], s_i = [7,9,18], d_j = [5,8,7,14]$
Problem 4 [30]	$c_{ij} = [32,40,120; 60,68,104; 200,80,60], s_i = [20,20,45], d_j = [30,35,30]$
Problem 5 [27]	$c_{ij} = [3,4,6; 7,3,8; 6,4,5; 7,5,], s_i = [100,80,90,120], d_j = [110,110,60]$
Problem 6 [35]	$c_{ij} = 3,6,1,5; 7,9,27; 2,4,2,1, s_i = [6,6,6], d_j = 4,5,4,5$
Problem 7 [34]	$c_{ij} = [1,2,3,4; 4,3,2,0; 0,2,2,1], s_i = [6,8,10], d_j = [4,6,8,6]$
Problem 8 [20]	$c_{ij} = [10,2,20,11; 12,7,9,20; 4,14,16,18], s_i = [15,25,10], d_j = [5,15,15,15]$
Problem 9 [2]	$c_{ij} = [2,1,3,2,2; 3,2,1,1,1; 5,4,2,1,3; 7,5,5,3,1], s_i = [20,70,30,60], d_j = [50,30,30,50,20]$

Appendix C

Problem chosen from Juman & Hoque [23]	Data of the problem
Problem 1	$c_{ij} = [6,8,10; 7,11,11; 4,5,12], s_i = [150,175,275], d_j = [200,100,300]$
Problem 2	$c_{ij} = [20,22,17,4; 24,37,9,7; 32,37,20,15], s_i = [120,70,50], d_j = [60,40,30,110]$
Problem 3	$c_{ij} = [4,6,8,8; 6.8.6.7; 5,7,6,8], s_i = [40,60,50], d_j = [20,30,50,50]$
Problem 4	$c_{ij} = [19,30,50,12; 70,30,40,60; 40,10,60,20], s_i = [7,10,18], d_j = [5,7,8,15]$
Problem 5[29]	$c_{ij} = [13,18,30,8; 55,20,25,40; 30,6,50,10], s_i = [8,10,11], d_j = [4,6,7,12]$
Problem 6	$c_{ij} = [25,14,34,46,45; 10,47,14,20,41; 22,42,38,21,46; 36,20,41,38,44], s_i = [27,35,37,45], d_j = [22,27,28,33,34]$
Problem 7	$c_{ij} = [9,12,9,6,9,10; 7,3,7,7,5,5; 6,5,9,11,3,11; 6,8,11,22,10], s_i = [150,175,275], d_j = [200,100,300]$

Appendix D

Problem chosen from Ahmed et al. [5]	Data of the problem
Problem 1	$c_{ij} = [3,1,7,4; 2,6,5,9; 8,3,3,2], s_i = [300,400,500], d_j = [250,350,400,200]$
Problem 2	$c_{ij} = [50,60,100,50; 80,40,70,50; 90,70,30,50], s_i = [20,38,16], d_j = [10,18,22,24]$
Problem 3	$c_{ij} = [7,5,9,11; 4,3,8,6; 3,8,10,5; 2,6,7,3], s_i = [30,25,20,15], d_j = [30,30,20,10]$
Problem 4	$c_{ij} = [4,3,5; 6,5,4; 8,10,7], s_i = [90,80,100], d_j = [70,120,80]$

Appendix E

Example 1(Balanced TP)

A company manufactures motor tyres and it has four factories F_1, F_2, F_3 and F_4whose weekly production capacities are 5,8,7 and 14 thousand pieces of tyres respectively. The company supplies tyres toits three showrooms located at D_1, D_2 and D_3 whose weekly demand are 7, 9 and 18 thousand pieces respectively. The transportation cost per thousand pieces of tyre is given below in the TT;

Factory	Showrooms			
	D_1	D_2	D_3	Supply
F_1	2	7	4	5
F_2	3	3	1	8
F_3	5	4	7	7
F_4	1	6	2	14
Demand	7	9	18	

We want to schedule the shifting of tyers from factories to showroom with a minimum cost.

Example 2(Unbalanced TP)

A company has four plants located at A,B,C and D, which supply to warehouses located at E,F,G,H and I. Monthly plant capacities are 300,500,825 and 375 units respectivelyand monthly warehouses requirements are 350,400,250,150 and 400 units respectively.Unit transportation costs are given below.

Plants	Warehouses					Capacities
	E	F	G	H	I	
A	10	2	16	14	10	300
B	6	18	12	13	16	500
C	8	4	14	12	10	825
D	14	22	20	8	18	375
Requirements	350	400	250	150	400	

Determine a distribution plan for the company in order to minimize the total transportation cost.

Appendix F

Validation of the new Algorithm via numerical example

Consider the following Transportation problem:

	D_1	D_2	D_3	D_4	Supply
S_1	60	120	75	180	8,000
S_2	58	100	60	165	9,200
S_3	62	110	65	170	6,250
S_4	65	115	80	175	4.900
S_5	70	135	85	195	6,100
Demand	5,000	2,000	10,000	6,000	

Step 1: Formulate the Transportation Cost Matrix. The problem is unbalanced, make it a balanced problem by introducing a dummy destination.

	D_1	D_2	D_3	D_4	D_5	**Supply**
S_1	60	120	75	180	0	8,000
S_2	58	100	60	165	0	9,200
S_3	62	110	65	170	0	6,250
S_4	65	115	80	175	0	4.900
S_5	70	135	85	195	0	6,100
Demand	5,000	2,000	10,000	6,000	11,450	

Step 2: Compute the path according to the probability matrix (using eq.(2)).

	D_1	D_2	D_3	D_4	D_5	**Supply**
S_1	.204	.194	.195	.197	0	8,000
S_2	.208	.219	.220	.210	0	9,200
S_3	.201	.205	.212	.204	0	6,250
S_4	.196	.200	.189	.201	0	4.900
S_5	.188	.179	.182	.185	0	6,100
Demand	5,000	2,000	10,000	6,000	11,450	

Step 3,4,5 and 6:

.204	~~.194~~	.195	.197	0	8,000
.208	.219*2000	.220	.210	0	7,200
.201	~~.205~~	.212	.204	~~0~~	6,250
.196	~~.200~~	.189	.201	0	4.900
.188	~~.179~~	.182	.185	0	6,100
5,000	~~2,000~~	10,000	6,000	11,450	

Step 3,4,5 and 6:

.204	~~.194~~	.195	.197	0	8,000
~~.208~~	.219*2000	.220*7,200	~~.210~~	~~0~~	~~7,200~~
.201	~~.205~~	.212	.204	~~0~~	6,250
.196	~~.200~~	.189	.201	0	4,900
.188	~~.179~~	.182	.185	0	6,100
5,000	0	2,800	6,000	11,450	

Step 3,4,5 and 6:

.204	~~.194~~	~~.195~~	.197	0	8,000
~~.208~~	.219***2,000**	.220***7,200**	.210	~~0~~	0
.201	~~.205~~	.212***2,800**	.204	~~0~~	3,450
.196	~~.200~~	~~.189~~	.201	0	4,900
.188	~~.179~~	~~.182~~	.185	0	6,100
5,000	0	~~2,800~~	6,000	11,450	

Step 3,4,5 and 6:

.204	~~.194~~	~~.195~~	.197	0	8,000
~~.208~~	.219***2,000**	.220***7,200**	~~.210~~	~~0~~	0
~~.201~~	~~.205~~	.212***2,800**	.204***3,450**	~~0~~	~~3,450~~
.196	~~.200~~	~~.189~~	.201	0	4,900
.188	~~.179~~	~~.182~~	.185	0	6,100
5,000	0	0	2,550	11,450	

Step 3,4,5 and 6:

.204	~~.194~~	~~.195~~	~~.197~~	0	8,000
~~.208~~	.219***2,000**	.220***7,200**	~~.210~~	~~0~~	0
~~.201~~	~~.205~~	.212***2,800**	.204***3,450**	~~0~~	0
.196	~~.200~~	~~.189~~	.201***2,550**	0	2,350
.188	~~.179~~	~~.182~~	~~.185~~	0	6,100
5,000	0	0	~~2,550~~	11,450	

Step 3,4,5 and 6:

.204***5000**	~~.194~~	~~.195~~	~~.197~~	0	3,000
~~.208~~	.219***2,000**	.220***7,200**	~~.210~~	~~0~~	0
~~.201~~	~~.205~~	.212***2,800**	.204***3,450**	~~0~~	0
~~.196~~	~~.200~~	~~.189~~	.201***2,550**	0	2,350
~~.188~~	~~.179~~	~~.182~~	~~.185~~	0	6,100
~~5,000~~	0	0	0	11,450	

Step 3,4,5 and 6:

.204***5000**	~~.194~~	~~.195~~	~~.197~~	0*3,000	~~3,000~~
~~.208~~	.219***2,000**	.220***7,200**	~~.210~~	~~0~~	0
~~.201~~	~~.205~~	.212***2,800**	.204***3,450**	~~0~~	0
~~.196~~	~~.200~~	~~.189~~	.201***2,550**	0*2,350	~~2,350~~
~~.188~~	~~.179~~	~~.182~~	~~.185~~	0*6,100	~~6,100~~
0	0	0	0	~~11,450~~	

Step 7: Stop and determine solution.

.

	D_1	D_2	D_3	D_4	**Supply**
S_1	60*5000	120	75	180	8,000
S_2	58	100*2000	60*7200	165	9,200
S_3	62	110	65*2800	170*3450	6,250
S_4	65	115	80	175*2550	4.900
S_5	70	135	85	195	6,100
Demand	5,000	2,000	10,000	6,000	

Total cost = 60x5000 + 100x2000 + 60x7200 + 65x2800 + 170x3450 + 175x2550 = 2,146,750

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