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On the exponential Diophantine equation $(2^{2m+1}-1)+(13)^n=z^2$

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Abstract: Nowadays, scholars are very interested to determine the solution of different Diophantine equations because these equations have numerous applications in the field of coordinate geometry, cryptography, trigonometry and applied algebra. These equations help us for finding the integer solution of famous Pythagoras theorem and Pell's equation. Finding the solution of Diophantine equations have many challenges for scholars due to absence of generalize methods. In the present paper, author studied the exponential Diophantine equation $(2^{2m+1}-1)+(13)^n=z^2$, where m,n are whole numbers, for its solution in whole numbers. Results show that the exponential Diophantine equation $(2^{2m+1}-1)+(13)^n=z^2$, where m,n are whole numbers, has no solution in whole number.

Keywords: Positive integer; Diophantine equation; Solution; Congruence; Modulo system.

1. Introduction

The class of Diophantine equations is classified in two categories, one is linear Diophantine equations and the other one is non-linear Diophantine equations. Both categories have numerous applications in solving the puzzle problems. Aggarwal *et al.*, [1] discussed the Diophantine equation $223^x+241^y=z^2$ for solution. Existence of solution of Diophantine equation $181^x+199^y=z^2$ was given by Aggarwal *et al.*, [2]. Bhatnagar and Aggarwal [3] proved that the exponential Diophantine equation $421^p+439^q=r^2$ has no solution in whole number.

Gupta and Kumar [4] gave the solutions of exponential Diophantine equation $n^x+(n+3m)^y=z^{2k}$. Kumar *et al.*, [5] studied exponential Diophantine equation $601^p+619^q=r^2$ and proved that this equation has no solution in whole number. The non-linear Diophantine equations $61^x+67^y=z^2$ and $67^x+73^y=z^2$ are studied by Kumar *et al.*, [6]. They determined that the equations $61^x+67^y=z^2$ and $67^x+73^y=z^2$ are not solvable in non-negative integers. The Diophantine equations $31^x+41^y=z^2$ and $61^x+71^y=z^2$ were examined by Kumar *et al.*, [7]. They proved that the equations $31^x+41^y=z^2$ and $61^x+71^y=z^2$ are not solvable in whole numbers.

Mishra *et al.*, [8] gave the existence of solution of Diophantine equation $211^\alpha+229^\beta=\gamma^2$ and proved that the Diophantine equation $211^\alpha+229^\beta=\gamma^2$ has no solution in whole number. Diophantine equations help us for finding the integer solution of famous Pythagoras theorem and Pell's equation [9,10]. Sroysang [11–14] studied the Diophantine equations $8^x+19^y=z^2$ and $8^x+13^y=z^2$. He determined that $\{x=1,y=0,z=3\}$ is the unique solution of the equations $8^x+19^y=z^2$ and $8^x+13^y=z^2$. Sroysang [12] studied the Diophantine equation $31^x+32^y=z^2$ and determined that it has no positive integer solution. Sroysang [13] discussed the Diophantine equation $3^x+5^y=z^2$.

Goel *et al.*, [15] discussed the exponential Diophantine equation $M_5^p+M_7^q=r^2$ and proved that this equation has no solution in whole number. Kumar *et al.*, [16] proved that the exponential Diophantine equation $(2^{2m+1}-1)+(6^{r+1}+1)^n=w^2$ has no solution in whole number. The exponential Diophantine equation $(7^{2m})+(6r+1)^n=z^2$ has studied by Kumar *et al.*, [17]. Aggarwal and Sharma [18] studied the non-linear Diophantine equation $379^x+397^y=z^2$ and proved that this equation has no solution in whole number. To determine the solution of exponential Diophantine equation $(2^{2m+1}-1)+(13)^n=z^2$, where m,n are whole numbers, in whole numbers is the main objective of this paper.

2. Preliminaries

Lemma 1. *For any whole number m, the exponential Diophantine equation $(2^{2m+1} - 1) + 1 = z^2$ is not solvable in whole number.*

Proof. For any whole number m, 2^{2m+1} is an even number so $(2^{2m+1} - 1) + 1 = z^2$ is an even number implies z is an even number. Which means

$$z^2 \equiv 0(mod3) \quad \text{or} \quad z^2 \equiv 1(mod3). \tag{1}$$

Now, for the same m, $2^{2m+1} \equiv 2(mod3)$ implies;

$$(2^{2m+1} - 1) + 1 = z^2 \equiv 2(mod3). \tag{2}$$

The result of (2) denies the result of (1), hence the exponential Diophantine equation $(2^{2m+1} - 1) + 1 = z^2$ is not solvable in whole number. □

Lemma 2. *For any whole number n, the exponential Diophantine equation $1 + (13)^n = z^2$ is not solvable in whole number.*

Proof. For any whole number n, $(13)^n$ is an odd integer, so $1 + (13)^n = z^2$ is an even integer, implies z is an even integer. Which means

$$z^2 \equiv 0(mod3) \quad \text{or} \quad z^2 \equiv 1(mod3). \tag{3}$$

Now, for the same n $13^n \equiv 1(mod3)$ implies;

$$1 + (13)^n = z^2 \equiv 2(mod3). \tag{4}$$

The result of (4) denies the result of (3), hence the exponential Diophantine equation $1 + (13)^n = z^2$ is not solvable in whole number. □

Theorem 1. *For any whole numbers m and n, the exponential Diophantine equation $(2^{2m+1} - 1) + (13)^n = z^2$ is not solvable in whole number.*

Proof. We have following four cases;

1. If $m = 0$ then the exponential Diophantine equation $(2^{2m+1} - 1) + (13)^n = z^2$ becomes $1 + (13)^n = z^2$, which is not solvable in whole numbers according to Lemma 2.
2. If $n = 0$ then the exponential Diophantine equation $(2^{2m+1} - 1) + (13)^n = z^2$ becomes $(2^{2m+1} - 1) + 1 = z^2$, which is not solvable in whole numbers according to Lemma 1.
3. If m, n are natural numbers then $(2^{2m+1} - 1), (13)^n$ are odd numbers, so $(2^{2m+1} - 1) + (13)^n = z^2$ is an even number, implies z is an even number. which means

$$z^2 \equiv 0(mod3) \quad \text{or} \quad z^2 \equiv 1(mod3). \tag{5}$$

   Now, $2^{2m+1} \equiv 2(mod3)$ and $(13)^n \equiv 1(mod3)$ implies

$$(2^{2m+1} - 1) + (13)^n = z^2 \equiv 2(mod3). \tag{6}$$

   The result of (6) denies the result of (5) hence the exponential Diophantine equation $(2^{2m+1} - 1) + (13)^n = z^2$, where m, n are positive integers, is not solvable in whole numbers.
4. If $m, n = 0$ then $(2^{2m+1} - 1) + (13)^n = 1 + 1 = 2 = z^2$, which is impossible because z is a whole number. Hence the exponential Diophantine equation $(2^{2m+1} - 1) + (13)^n = z^2$, where $m, n = 0$ is not solvable in whole numbers.

□

3. Conclusion

In this paper, author fruitfully studied the exponential Diophantine equation $(2^{2m+1} - 1) + (13)^n = z^2$, where m, n are whole numbers, for its solution in whole number. Author determined that the exponential Diophantine equation $(2^{2m+1} - 1) + (13)^n = z^2$, where m, n are whole numbers, is not solvable in whole numbers.

Conflicts of Interest: "The author declares no conflict of interest."

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