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Abstract Geometry and Its Applications in Quantum Mechanics

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Abstract

We examine a series of developments in geometry. These include the Theorem of Bézout. We then examine how several developments in geometry can be used in application to Quantum mechanics.

Keywords

Rings, Polynomials, Theorem of Bézout, Quantum Mechanics

1. Introduction

Abstract geometry simply means the geometry of a space, or spaces, that are different from Eucliden geometry. An example of such considerations is (Allan, 2011). Let us begin by considering a quite fundamental theorem, named after Bézout (Bézout, 2006); how it may be formulated and proven. This theorem is normally set in a context of geometry and algebra. The geometry, projective geometry, was described by two authors (Veblen & Young, 1938). This book is a classic and a model of mathematical exposition. The projective geometry that it describes is quite different from Euclidean geometry. For this reason, it is instructive and improves the intuition, to consider models of projective geometry, such as those presented in (Apéry, 1987). The algebraic context that is often used for the theorem of Bézout was provided to us by Emmy Noether, one of the most brilliant female mathematicians of all time. She escaped from the rising power of Hitler's dictatorship in Germany to take a passenger liner to America. There she lectured at Bryn Mawr. Of her many contributions to mathematics, (Noether, 1921) is the most relevant for us here.

Readers who may wish to refresh their memory of polynomial, and other algebraic, curves, leading up to the Theorem of Bézout, may consult (Gibson, 1998).

1) The Theorem of Étienne Bézout and its proof.

2) Let P and P' be two homogeneous polynomials, in the plane, of degree x and y respectively. Let P and P' have no commom factors.

3) Theorem of Bézout: $0 \le I(P, P') \le xy$ where I is the intersection multiplicity.

4) *Proof.*

5) Corollaries of Commutative Algebra, (Kemper, 2011) is a standard text.

6) Corollaries of Algebraic Geometry, (Hulek, 2003) is recommended as a standard text.

7) Topics 5 and 6 above cover the elements of algebra that are required to arrive at a complete formulation of a proof of the Theorem. There is another signifičnt point that any such proof must cover. It is intersection multiplicity.

8) (Brieskorn & Knorrer, 1986) offers a completly self-contained and detailed proof of the Theorem of Bézout

9) *QED.*

10) Refinements: Projective Geometry, in (Veblen & Young, 1938), Differential Geometry, in (O'Neill, 1966), Zariski Topology in (Kemper, 2011, Chapter 3).

2. Applications

2.1. Quantum Mechanics

Fiber optic cables extend throughout our world. Many automatic processes are guided by computer programs stored on semiconductor microchip computers. These are examples of applications of quantum mechanics. The logical structure of the these quantum processes are explored in (Bub, 1998).

In addition to such very practical matters as those above, quantum mechanics also has connections to cosmology. One of the most significant of such discoveries of the 20th Century was Hawking radiation from black holes. The characteristics of this radiation were derived by Stephen Hawking from principles of quantum mechanics.

The first impressions of students who begin to study quantum mechanics are often counter-intuitive. How these first impressions may be dispelled by further study is discussed in (Dickson, 1997).

2.2. Games and Quantum Mechanics

Many were recently saddened by the news of the death of the mathematician John Horton Conway (1937-2020). He was not only a mathematician of high excellence, but also enjoyed offering playful expositions of mathematics to those whom he encountered. He invented a mathematical game that he called Life. This game was played on an array of cells and was described by Martin Gardner in (Gardner, 1970).

A series of scientists, beginning with John von Neumann and Stanislaw Ulam, foresaw that the growth of patterns in such cells could shed light upon the interpretation of Quantum Theory. The history of this development, with literature references, is contained in (t'Hooft, 2016: pp. 14-17).

2.3. Cellular Automata

The configurations of occupied cells that emerge as the game of Life is played are indeed, as Martin Gardner expresses it, fantastic. They also have a much deeper significnce. Gerard t'Hooft, Nobel Prize laureate, was awarded the Nobel Prize in Physics, 1999. As Gerard t'Hooft shows us, in (t'Hooft, 2016: pp. 106-108), Hawking radtiation can be computed using methods that rely upon cellular automata.

3. Findings and Results

We have described some findings and some initial results. The reader who is interested in greater deail in these sections may wish to read (Jost, 2008). The author, who is Codirector of the Max Planck Institute for Mathematics in the Sciences in Leipzig, has dedicated this book to "Shing-Tung Yau for so many discussions about mathematics and Chinese culture".

4. Conclusion

Geometry and other parts of mathematics have significant applications in modern quantum mechanics. These applications have already brought important advances in quantum mechanics. We hope, in a follow-on publication, to explore in detail the contributions to modern quantum mechanics made by the mathematicians Emmy Noether and John Horton

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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