*American Chemical Science Journal*
*4(6): 901-917, 2014*

SCIENCEDOMAIN *international*
www.sciencedomain.org

# Unsteady MHD Heat and Mass Transfer Flow over Stretching Sheet in Porous Medium with Variable Properties Considering Viscous Dissipation and Chemical Reaction

**D. Hunegnaw[1*] and N. Kishan[1]**

[1]*Department of Mathematics, Osmania University, Hyderabad 500007, Andhra Pradesh, India.*

***Authors' contributions***

*This work was carried out in collaboration between the both authors DH and NK. Author NK designed the study, wrote the protocol and wrote the first draft of the manuscript. Author DH deals with the calculation part, managed the analysis of the study and executed the program. Both authors read and approved the final manuscript.*

***Original Research Article***

***Received 11th June 2014***
***Accepted 9th July 2014***
***Published 25th July 2014***

## ABSTRACT

Unsteady magneto hydrodynamic heat and mass transfer flow of viscous, incompressible fluid flow over a stretching sheet embedded in a porous medium with variable viscosity and thermal conductivity have been analyzed, by taking into account viscous dissipation and chemical reaction. The governing coupled non-linear partial differential equations are reduced to similarity boundary layer equations using suitable transformation and then solved using the Runge-Kutta fourth order procedure along with shooting technique. The effects of various flow parameters on the velocity, temperature and concentration distributions are analyzed and presented graphically. Skin-friction coefficient, Nusselt number and Sherwood number are derived, discussed numerically and their numerical values for various values of physical parameter are presented through tables. It is noted that the skin-friction coefficient $-f''(0)$ and temperature gradient $-\theta'(0)$ increase with the increase of variable viscosity as well as velocity suction parameter but the Sherwood number $-\varphi'(0)$ decreases. The effect of Eckert number and Prandtl number increases the values of $-f''(0)$ and mass transfer rate $-\varphi'(0)$ but decrease the temperature gradient $-\theta'(0)$.

**Corresponding author: Email: hunegnawd@yahoo.com;*

*Keywords: MHD; unsteady; chemical reaction; viscous dissipation; variable viscosity; variable thermal conductivity.*

## 1. INTRODUCTION

The magneto hydrodynamic boundary layer flow of an incompressible and electrically conducting fluid is encountered in geophysics, astrophysics and in many engineering and industrial processes. The magneto hydrodynamics heat and mass transfer flow in the boundary layer induced by a moving surface in a fluid finds important applications in chemical engineering and electronics, meteorology and metallurgy etc. The study of boundary layer flow over continuous solid surface moving with constant velocity in an ambient fluid was initiated by Sakiadis [1]. Erickson et al. [2] extended Sakiadis [1] problem to include blowing or suction at the moving surface. Subsequently Tsou et al. [3] presented a combined analytical and experimental study of the flow and temperature fields in the boundary layer on a continuous moving surface. R. Ellahi et al. [4] investigated numerical analysis of steady flows with heat transfer, MHD and nonlinear slip effects. Nadeem et al. [5] studied the numerical study of MHD boundary layer flow of a Maxwell fluid past a stretching sheet in the presence of nanoparticles. Nadeem et al. [6] investigated the MHD three dimensional Casson fluid towards past a porous linearly stretching sheet. Chakrabarti et al. [7] studied the temperature distribution in MHD boundary layer flow over a stretching sheet in the presence of suction. Abdul et al. [8] studied heat transfer analysis of the boundary layer flow over a vertical exponential stretching cylinder. Mixed convection heat transfer in micropolar nanofluid over a vertical slender cylinder is investigated by Abdul et al [9]. Akbar et al. [10] investigated the numerical solutions of magnetohydrodynamic boundary layer flow of tangent hyperbolic fluid towards a stretching sheet. Simultaneous heat and mass transfer in porous media with different geometries has attracted many researchers due to its numerous engineering and geophysical applications such as geothermal reservoirs, thermal insulation, packed-bed catalytic reactors, underground energy transport, cooling of nuclear reactors, enhanced oil recovery, drying of porous solids, etc. Seddeek et al. [11] studied the effects of the chemical reaction and variable viscosity on hydromagnetic mixed convection heat and mass transfer for the Hiemenz flow through porous media with radiation. Ghay et al. [12] applied the Chebyshev finite difference method to study the effects of the chemical reaction and heat and mass transfer on a laminar flow with variable viscosity along a semi-infinite horizontal plate. Zefar et al. [13] studied the effect of variable thermal conductivity on heat transfer from a hollow sphere with heat generation using homotopy perturbation method. Rahman et al. [14] investigated the hydromagnetic heat and mass transfer flow over an inclined heated surface, taking account variable viscosity and electric conductivity. Gupta et al. [15] studied the heat and mass transfer on a stretching sheet subject to suction or blowing. Seddek et al. [16] studied the effects of the variable viscosity and thermal conductivity effects on an unsteady two-laminar MHD flow past a vertical porous moving plate considering the effect of variable suction. The effects of variable viscosity on the flow of non-Newtonian fluid through a porous medium in an inclined channel with slip conditions are investigated by Ambreen et al. [17].Salem [18] investigated the variable viscosity and thermal conducting effects on MHD flow heat transfer in viscoelastic fluid over a stretching sheet. Hassan et al. [19] studied the effects of MHD and temperature dependent viscosity on the flow of non-Newtonian nanofluid in pipe. Hassnian [20] investigated unsteady non-Darcy mixed convection flow near the stagnation point on a heated vertical surface embedded in a porous medium with thermal radiation and variable viscosity. Conjugated forced convection heat transfer from a heated flate plate of finite thickness and temperature dependent thermal conductivity is considered by Mohammed et al. [21].

Chemical reaction can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. The effect of the presence of foreign mass on the free convection flow past a semi-infinite vertical plate was studied by Gebhart et al. [22]. The presence of a foreign mass in air or water causes some kind of chemical reaction. During a chemical reaction between two species, heat is also generated (Byron Bird R.et al. [23]). In most of cases of chemical reaction, the reaction rate depends on the concentration of the species itself. A reaction is said to be first order if the rate of reaction is directly proportional to concentration itself (Cussler [24]). Shakhaoath et al. [25] studied the possessions of chemical reaction on MHD Heat and mass transfer of nanofluid flow on a continuously moving surface. Chemical reaction effects on heat and mass transfer laminar boundary layer flow have been discussed by many authors (Apelblat [26]; Das et al. [27]; Muthucumaraswamy et al. [28]) in various situations. Viscous dissipation effects are important in geophysical flows and also in certain industrial operations and are usually characterized by the Eckert number. Viscous dissipation changes the temperature distributions by playing a role like energy source, which leads to affect heat transfer rates. The merit of the effect of viscous dissipation depends on whether the sheet is being cooled or heated. Mahajan et al. [29] reported the influence of viscous heating dissipation effects in natural convective flows, showing that the heat transfer rates are reduced by an increase in the dissipation parameter. Mohammadi et al. [30] studied the analytical solution for two-phase flow between two rotating cylinders filled with power law liquid and a micro layer of gas. Gebhart et al. [31] considered the effects of viscous dissipation for external natural convection flow over a surface. Soundalgekar [32] analyzed viscous dissipative heat on the two-dimensional unsteady free convective flow past an infinite. Israel Cookey et al. [33] investigated the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Goverdhan et al. [34] studied effect of viscous dissipation and radiation on MHD gas flow and heat and mass transfer over a stretching sheet surface with a uniform free stream. Ramachandra et al. [35] presented radiation and mass transfer effects on an unsteady MHD free convection flow past a heated vertical plate with viscous dissipation.

Although there are numerous widely practical applications in industrial processes, few previous published papers discussed the combined relation. In the present paper, we make attempt to investigate the problem of heat and mass transfer of incompressible unsteady MHD flow fluid embedded in a porous medium over a stretching sheet with variable viscosity and thermal conductivity under a chemical reaction and viscous dissipation. The presence of combined buoyancy effects leads to the momentum, heat and mass transfer equations in the coupled form of highly non-linear partial differential equations. The equations (13)-(15) together with boundary conditions (16) can be solved by semi-analytical means introduced in recent years such as homotopy, DTM and ADM which are explained very well by Mohammed et al. [36] and Ali et al. [37]. However in this paper a numerical shooting technique for three unknown initial conditions with Runge-Kutta fourth order integration scheme has been developed. The results are analyzed for various physical parameters such as variable viscosity and porous medium parameter, unsteadiness parameter, magnetic parameter, thermal conductivity parameter, chemical reaction parameter, Eckert number, Schmidt number and velocity suction parameter on the flow, heat and mass transfer characteristics.

## 2. FORMULATION OF THE PROBLEM

Consider unsteady-two-dimensional MHD forced convection laminar and stable boundary-layer flow of viscous dissipating incompressible fluid past a heated stretching sheet embedded in a porous medium and moving with variable velocity $u_w$. The surface (wall) temperature $T_w$ is held uniform and higher than the ambient temperature$T_\infty$. It is assumed that all body forces except magnetic field are neglected. The fluid viscosity and thermal diffusivity are assumed to vary as linear functions of temperature. A uniform transverse magnetic field is applied parallel to the direction of flow and chemical reaction is taking place in the flow. Under the usual boundary-layer approximation, the governing equations for the flow and heat and mass transfer have the form

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\mu\frac{\varphi}{k}u-\frac{\sigma {B_0}^2}{\rho}u \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y} = \frac{1}{\rho c_p}\frac{\partial}{\partial y}(k^*\frac{\partial T}{\partial y})+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2}-{k_1}^*(C-C_\infty) \tag{4}$$

where $u$ and $v$ are the velocity components along the $x$ and $y$ coordinates ,respectively, $\mu$ is the coefficient of viscosity, $\rho$ is the mass density of the fluid , $c_p$ is the specific heat at constant pressure, $\varphi$ is the porosity, $k$ is the permeability, $k^*$ is the thermal conductivity, $D_m$ is the molecular diffusivity, ${k_1}^*$ is chemical reaction parameter, and $T$ and $C$ are the temperature and concentration of the species, respectively.

The corresponding boundary conditions are:

$$u=u_w(x,t)=cx/(1-\alpha t),\ v=v_w(t),\ T=T_w(x,t),\ C=C_w(x,t) \quad at \quad y=0.$$

$$u=0, \quad T=T_\infty, \quad C=C_\infty \qquad at\ y\to\infty \tag{5}$$

where $v_w=-v_0\sqrt{v^*c/(1-\alpha t)}$ is the fluid suction velocity, $c$ is the initial stretching rate, $\alpha$ is a constant and $v^*$ is the kinematic viscosity of the ambient fluid: the subscripts $w$ and $\infty$ refer to the plate and boundary–layer edge respectively.

Following Abdou [38], we assume that the sheet temperature $T_w(x,t)$ and sheet concentration $C_w(x,t)$ have the following form

$$T_w(x,t)=T_\infty+\frac{c}{2vx^2}(1-\alpha t)^{-\frac{3}{2}},\ C_w(x,t)=C_\infty+\frac{c}{2vx^2}(1-\alpha t)^{-\frac{3}{2}} \tag{6}$$

We introduce the following relations and dimensionless variables

$$u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x},\ \eta=\sqrt{\frac{c}{v^*(1-\alpha t)}}y, \quad \psi=\sqrt{\frac{v^*c}{(1-\alpha t)}}xf(\eta)$$

$$T = T_\infty + \frac{c}{2v^* x^2}(1-\alpha t)^{-\frac{3}{2}}\theta(\eta),\ C = C_\infty + \frac{c}{2v^* x^2}(1-\alpha t)^{-\frac{3}{2}}\varphi(\eta) \tag{7}$$

and

$$\theta = \frac{\mathrm{T}-\mathrm{T}_\infty}{\mathrm{T_W}-\mathrm{T}_\infty},\ \varphi = \frac{\mathrm{C}-\mathrm{C}_\infty}{\mathrm{C_W}-\mathrm{C}_\infty} \tag{8}$$

where $\psi$ is the stream function satisfying the continuity equation, $f(\eta)$ is the dimensionless stream function, $\eta$ is pseudo-similarity variable, $\theta(\eta)$ is the dimensionless temperature of the fluid in the boundary layer region, $v^* = \mu_\infty/\rho$ is the free stream kinematic viscosity.

Following Ling et al. [39] and Lai et al. [40], we take the temperature dependent viscosity of the form

$$\mu = \frac{\mu_\infty}{[1+a(T-T_\infty)]} \tag{9}$$

where $\mu_\infty$ is a constant value of the coefficient of viscosity far away from the sheet and $a > 0$ is a constant. Note that the dimensionless temperature $\theta$ can also be written as

$$\theta = \frac{T-T_r}{T_W-T_\infty} + \theta_r,$$
$$\theta_r = \frac{T_r-T_\infty}{T_W-T_\infty}, \quad T_r = T_\infty - \frac{1}{a} \tag{10}$$

where $\theta_r$ is the variable viscosity parameter. Substituting Eq.(10) into Eq.(9), we immediately find

$$\mu = \mu_\infty \frac{\theta_r}{\theta_r-\theta} \tag{11}$$

The variation of the thermal conductivity $k^*$ is taken of form as (Husain et al. [41])

$$k^* = k_\infty(1+\beta\theta) \tag{12}$$

where $k_\infty$ is the value of the thermal diffusivity at the surface of temperature $T_w$ and $\beta$ is a parameter that depends on the nature of the fluid.

Substituting Eqs (8) and (9) into Eqs(2),(3) and (4) , we obtain the ordinary differential equations

$$f''' = A\left(1-\frac{\theta}{\theta_r}\right)\left(\frac{\eta}{2}f'' + f'\right) + \left(1-\frac{\theta}{\theta_r}\right)f'^2 - \left(1-\frac{\theta}{\theta_r}\right)ff''$$
$$-\frac{\theta'}{\theta_r-\theta}f'' + M\left(1-\frac{\theta}{\theta_r}\right)f' + kf' \tag{13}$$

$$\theta'' + \beta\theta\theta'' + \beta\theta'^2 = \Pr\left(1-\frac{\theta}{\theta_r}\right)\left(\frac{A}{2}\eta\theta' + \frac{3}{2}A\theta + 2f'\theta - f\theta'\right) - PrEcf''^2 \tag{14}$$

$$\varphi'' = Sc\left(\frac{A}{2}\eta\varphi' - f\varphi' + 2f'\varphi + k_1{}^*\phi\right) \tag{15}$$

and the corresponding boundary conditions Eqs(6) in the dimensionless form

$$f' = 1,\ f = f_w, \quad \theta = 1, \quad \varphi = 1 \qquad \text{at} \quad \eta = 0$$

$$f^{'} = 0 \quad \theta = 0, \quad \varphi = 0 \qquad\qquad \text{at} \quad \eta \to \infty \tag{16}$$

where $A = \frac{\alpha}{c}$ is the unsteadiness parameter, $k = \left(\frac{\varphi}{k}\right)(v^*/c(1-\alpha t))$ is the porous medium parameter, $Pr = \left(\frac{\mu}{k}\right)c_p = \frac{\theta_r}{(\theta_r - \theta)}Pr_\infty$ is the Prandtl number, $Sc = {v^*}/{D_m}$ is the Schmidt number, $M = \frac{\sigma {B_0}^2}{\rho c}(1-\alpha t)$ is magnetic parameter, $k_1 = \frac{{k_1}^*(1-\alpha t)}{c}$ is chemical reaction parameter, $Ec = \frac{{u_w}^2}{c_p(T_w - T_\infty)}$ is Eckert number and $f_w$ is the suction parameter.

The quantities of practical interest are the local skin-friction $C_f$, the local Nusselt number $N_{u_x}$ and the local Sherwood number $Sh_x$ which are defined as

$$C_f = \frac{\tau_w}{(\rho {u_w}^2)}, N_{u_x} = \frac{x q_w}{k^*(T_w - T_\infty)}, \quad Sh_x = \frac{x q_m}{D_m(C_w - C_\infty)}$$

where $k^*$ is the thermal conductivity of the fluid, $\tau_w, q_w$ and $q_m$ are, respectively, given by

$$\tau_w = -\mu\left(\frac{\partial u}{\partial y}\right)_{y=0,} \; q_w = -k^*\left(\frac{\partial T}{\partial y}\right)_{y=0,} \qquad q_m = -D_m\left(\frac{\partial T}{\partial y}\right)_{y=0,}$$

Then the local skin-friction coefficient, local Nusselt number and local Sherwood number are, respectively, given by

$$C_f {Re_x}^{1/2} = -f''(0), \; {Re_x}^{-1/2} N_{u_x} = -\theta^{'}(0), \quad {Re_x}^{-1/2} Sh_x = -\phi^{'}(0)$$

where $Re_x = \frac{U_w x}{v}$ is the local Reynolds number.

## 3. SOLUTION OF THE PROBLEM

In this section, we present the numerical solution of the system of ordinary equations (13)-(15) subject to the boundary conditions (16). Equations (13)-(15) constitute a highly non-linear coupled boundary value problem of third and second order. So we develop most effective numerical shooting technique with fourth-order-Runge-Kutta integration scheme with Newton Raphson Method. To select $\eta_\infty$ we begin with some initial guess value and solve the problem with some particular set of parameters to obtain $f^{''}(0)$, $\theta^{'}(0)$, and $\varphi^{'}(0)$. The solution process is repeated with another larger value of $\eta_\infty$ until two successive values of $f^{''}(0)$, $\theta^{'}(0)$, and $\varphi^{'}(0)$ differ only after desired digit signifying the limit of the boundary along $\eta$. The last value of $\eta_\infty$ is chosen as appropriate values for that particular set of parameters. Equations (13)-(15) has been reduced to a system of seven simultaneous equations of first order for seven unknowns following the method of superposition [42]. To solve this system, we require seven initial conditions whereas we have only two initial conditions $f^{'}(0)$ and $f(0)$ on $f$, one initial on each $\theta$ and $\varphi$, namely $\theta(0)$ and $\varphi(0)$ respectively. But the other three initial conditions $f^{''}(0)$, $\theta^{'}(0)$ and $\varphi^{'}(0)$ are not prescribed. Now, we employ numerical shooting technique where these three ending boundary conditions are utilized to produce three known initial conditions at $\eta$=0. The resultant initial value problem is then solved by employing Runge-Kutta fourth order method. The step size $\Delta\eta = 0.001$ is used to obtain the numerical solution with five decimal accuracy as criterion of convergence.

## 4. RESULTS AND DISCUSION

In order to get a clear insight of the physical problem, numerical computations have been carried out using fourth order Runge -Kutta method along with shooting technique for various values of different parameters such as unsteadiness parameter A, porous medium parameter K, Prandtl number Pr, magnetic parameter $M,$ chemical reaction parameter $k_1$, Eckert number $Ec$ , Schmidt number Sc and velocity suction parameter $f_w$. In order to verify the validity and accuracy of the present analysis, the results for the skin-friction$-f''(0)$, heat transfer $-\theta'(0)$ and mass transfer $-\varphi'(0)$ were compared with those reported by S. Husnain et al. [41]. The comparison in the above cases is found to be in excellent agreement as shown in Table 1. The values of local skin-friction coefficient , temperature gradient and mass transfer rate are tabulated in Table 2. It is noted that from Table 2, both the values of skin-friction $-f''(0)$ and temperature gradient $-\theta'(0)$ increase with the increase of variable viscosity as well as velocity suction parameter but the Sherwood number $-\varphi'(0)$ decreases. From the table, it is observed that the temperature gradient $-\theta'(0)$ and the Sherwood number $-\varphi'(0)$ decreases with the increase of unsteadiness parameter and the variable thermal conductivity whereas the skin-friction coefficient $-f''(0)$ increases. It is noted that the increase of magnetic parameter is to increase the skin –friction coefficient $-f''(0)$ and decrease the temperature gradient $-\theta'(0)$ and the Sherwood number $-\varphi'(0)$. Both the skin-friction coefficient $-f''(0)$ and the mass flow rate $-\varphi'(0)$ decrease with the increase of Prandtl number but the Nusselt number $-\theta'(0)$ increases. From the same table, it is clearly noted that the increase of the Eckert number is to increase the skin-friction coefficient $-f''(0)$ and the mass flow rate $-\varphi'(0)$ but to decrease the temperature gradient$-\theta'(0)$. Finally, the mass flow rate increases with the increase of chemical reaction parameter but decreases with the increase of Schmidt number as it is noticed from Table 2.

**Table 1. Comparison of S. Husnain [41] and the present study for M=0.5, k=0.5, $f_w$=0 and Sc=2**

| $\beta$ | Pr | S. Husnain [31] | | | Present study | | |
|---|---|---|---|---|---|---|---|
| | | $-f''(0)$ | $-\theta'(0)$ | $-\varphi'(0)$ | $-f''(0)$ | $-\theta'(0)$ | $-\varphi'(0)$ |
| 0.5 | 0.5 | 0.9624 | 0.6169 | 1.8948 | 0.9624 | 0.6169 | 1.8938 |
| 0.7 | 0.5 | 0.9690 | 0.5644 | 1.8950 | 0.9691 | 0.5644 | 1.8939 |
| 1.0 | 0.5 | 0.9774 | 0.5035 | 1.8947 | 0.9775 | 0.5034 | 1.8938 |
| 1.5 | 0.5 | 0.9885 | 0.4314 | 1.8943 | 0.9885 | 0.3414 | 1.8934 |
| 0.5 | 0.7 | 0.9447 | 0.7560 | 1.8925 | 0.9447 | 0.7560 | 1.8938 |
| 0.5 | 1.0 | 0.9249 | 0.9352 | 1.8925 | 0.9249 | 0.9354 | 1.8922 |
| 0.5 | 1.5 | 0.9017 | 1.1852 | 1.8912 | 0.9018 | 1.1852 | 1.8937 |
| 0.5 | 2.0 | 0.8852 | 1.3973 | 1.8887 | 0.8853 | 1.3974 | 1.8916 |

The dimensionless velocity, temperature and concentration profiles are shown graphically in the Figs. 1-10 for the different flow parameters. For different values of the Prandtl number, the velocity and the temperature profiles are plotted in Figs. 1a and 1b respectively. From Fig 1a, it is clear that an increase in the Prandtl number leads to a fall in the velocity. The temperature decreases with the increase of Pr as it can be seen from Fig. 1b. An increase in Prantdl number reduces the thermal boundary layer thickness. Prandtl number signifies the ratio of momentum diffusivity to thermal diffusivity. In heat transfer problems, the Prandtl number Pr controls the relative thickening of the momentum and thermal boundary layers. When Prandtl number Pr is small, heat diffuses quickly compared to the velocity (momentum), which means that for liquid metals, the thickness of the thermal boundary layer is much bigger than the momentum boundary layer. Fluids with lower Prandtl number have higher thermal conductivities (and thicker thermal boundary layer structure) so that heat can diffuse from the sheet faster than for higher Pr fluids (thicker boundary layers). Hence Prandtl number can be used to increase the rate of cooling in conducting flows. Effects of variable viscosity parameter $\theta_r$ on the velocity, temperature and concentration profiles are clearly exhibited in Figs. 2a, 2b and 2c respectively. From Fig. 2a and 2b, it is observed that fluid velocity decreases with increasing values of viscosity parameter but the temperature increases in this case. This is due to the fact that increasing the viscosity parameter leads to increase in the skin-friction coefficient which causes a decrease in the velocity of the fluid. Physically, the thermal viscosity cause a rise in friction, when friction increases, the area of the stretching surface in contact with the flow increases, therefore generated heat from the friction on the surface is transferred to the flow. This leads to arise in the surface temperature and the flow is heated. From Fig. 2c, it is observed that an increase of variable viscosity is to increase the concentration profiles. Figs. 3a-3c shows the effect of the unsteadiness parameter A on the velocity, temperature and concentration profiles respectively.

**Table 2. Numerical values of the skin-friction, Nusselt number and Sherwood number for different values of Pr, $\theta_r$, A, k, $f_w$, β, M, Ec, $k_1$ and Sc**

| $\theta_r$ | A | k | $f_w$ | β | Pr | M | Ec | $k_1$ | Sc | $-f''(0)$ | $-\theta'(0)$ | $-\varphi'(0)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 0.5 | 0.5 | 1 | 0.5 | 0.71 | 0.5 | 0.03 | 0 | 2 | 1.315521 | 0.905605 | 3.406687 |
| 2 | 0.5 | 0.5 | 1 | 0.5 | 3 | 0.5 | 0.03 | 0 | 2 | 1.210517 | 2.584057 | 3.391730 |
| 2 | 0.5 | 0.5 | 1 | 0.5 | 7 | 0.5 | 0.03 | 0 | 2 | 1.154246 | 4.938373 | 3.372522 |
| 2 | 0.5 | 0.5 | 1 | 0.5 | 0.71 | 0.5 | 0.5 | 0 | 2 | 1.326698 | 0.760853 | 3.164172 |
| 2 | 0.5 | 0.5 | 1 | 0.5 | 0.71 | 0.5 | 2 | 0 | 2 | 1.362411 | 0.318369 | 3.168962 |
| 2 | 0.5 | 0.5 | 1 | 0.5 | 0.71 | 0.5 | 0.03 | 0.5 | 2 | 1.315521 | 0.905605 | 3.406687 |
| 2 | 0.5 | 0.5 | 1 | 0.5 | 0.71 | 0.5 | 0.03 | 2 | 2 | 1.315521 | 0.905605 | 3.996270 |
| 4 | 0.5 | 0.5 | 1 | 0.5 | 0.71 | 0.5 | 0.03 | 0 | 2 | 1.732759 | 1.021184 | 3.0934 |
| 6 | 0.5 | 0.5 | 1 | 0.5 | 0.71 | 0.5 | 0.03 | 0 | 2 | 1.860852 | 1.058091 | 3.072680 |
| 2 | 1 | 0.5 | 1 | 0.5 | 0.71 | 0.5 | 0.03 | 0 | 2 | 1.372704 | 1.015237 | 3.013987 |
| 2 | 1.5 | 0.5 | 1 | 0.5 | 0.71 | 0.5 | 0.03 | 0 | 2 | 1.429527 | 1.111628 | 2.784796 |
| 2 | 0.5 | 1 | 1 | 0.5 | 0.71 | 0.5 | 0.03 | 0 | 2 | 1.474712 | 0.885879 | 3.129954 |
| 2 | 0.5 | 2 | 1 | 0.5 | 0.71 | 0.5 | 0.03 | 0 | 2 | 1.756829 | 0.857967 | 3.072787 |
| 2 | 0.5 | 0.5 | 1 | 1 | 0.71 | 0.5 | 0.03 | 0 | 2 | 1.333986 | 0.721524 | 3.164616 |
| 2 | 0.5 | 0.5 | 1 | 2 | 0.71 | 0.5 | 0.03 | 0 | 2 | 1.358428 | 0.529516 | 3.165960 |
| 2 | 0.5 | 0.5 | 1 | 0.5 | 0.71 | 1 | 0.03 | 0 | 2 | 1.415051 | 0.893204 | 3.140282 |
| 2 | 0.5 | 0.5 | 1 | 0.5 | 0.71 | 5 | 0.03 | 0 | 2 | 2.032050 | 0.830874 | 3.007053 |
| 2 | 0.5 | 0.5 | 0 | 0.5 | 1 | 0.5 | 0.22 | 0.5 | 1.5 | 1.043704 | 0.851642 | 1.835546 |
| 2 | 0.5 | 0.5 | 0 | 0.5 | 1 | 0.5 | 0.22 | 0.5 | 3 | 1.043704 | 0.851642 | 2.703179 |
| 2 | 0.5 | 0.5 | 0.2 | 0.5 | 1 | 0.5 | 0.22 | 0.5 | 2 | 1.089311 | 0.890256 | 2.372535 |

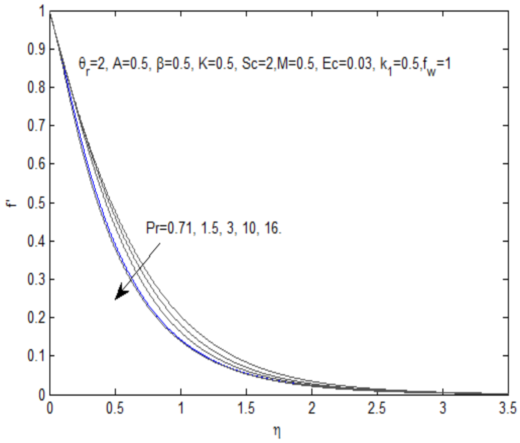


**Fig. 1a. Velocity for different values of Prandtl number Pr**

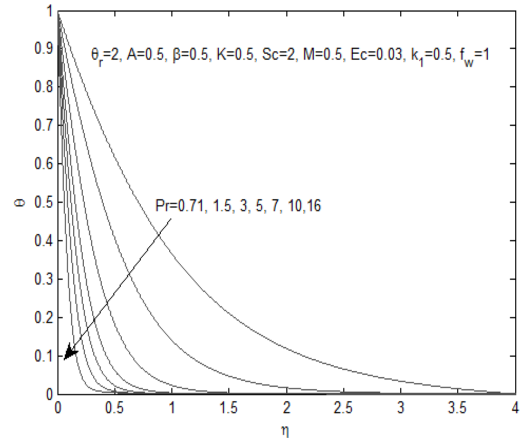


**Fig. 1b. Temperature profiles for different values of Prandtl number Pr**

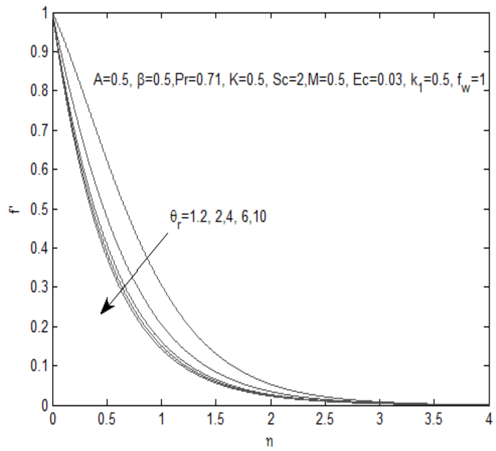


**Fig. 2a. Velocity profiles for different values of variable viscosity $\theta_r$**

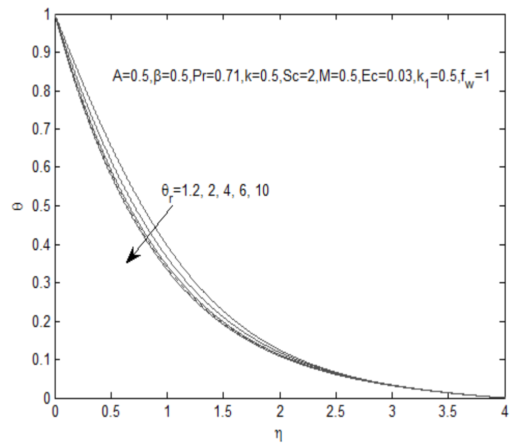


**Fig. 2b. Temperature profiles for different values of variable viscosity $\theta_r$**

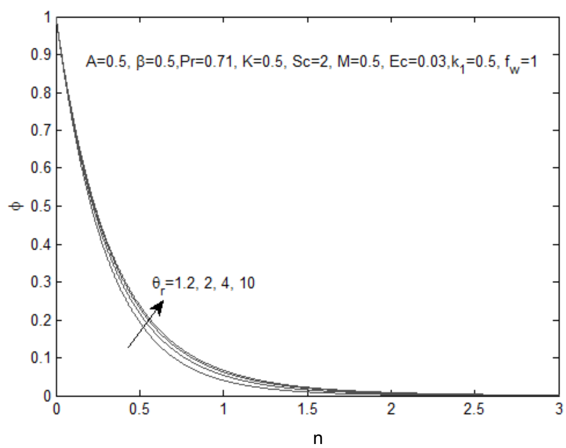


**Fig. 2c. Concentration profiles for different values of variable viscosity $\theta_r$**

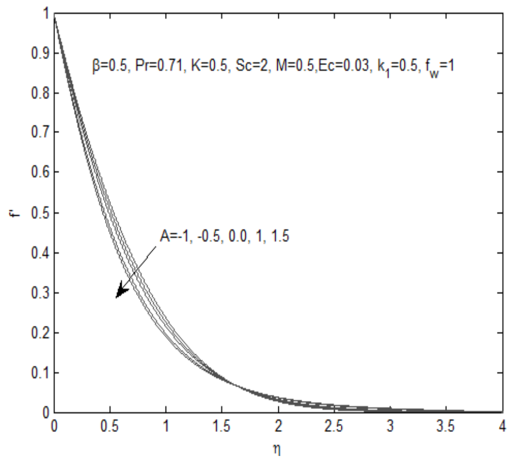


**Fig. 3a. Velocity profiles for different values of unsteadiness parameter A**

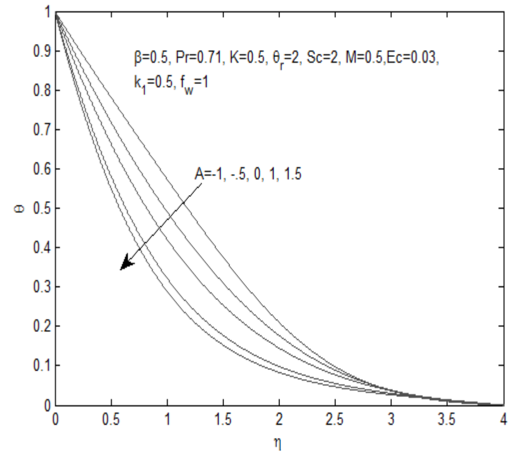


**Fig. 3b. Temperature for different values of unsteadiness parameter A**

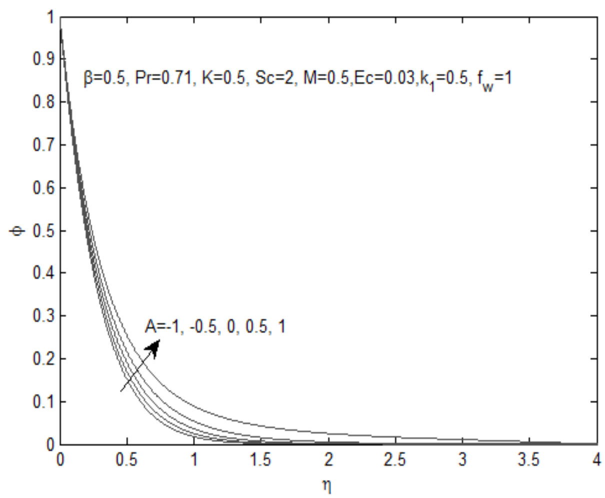


**Fig. 3c. Concentration profiles for different values of unsteadiness parameter A**

One can note that the increase of A results in decreasing the velocity, the temperature and the concentration of the fluid. So the increase of A leads to thinning of the velocity, the temperature and the concentration boundary layers. The variation of the concentration profiles with the chemical reaction $k_1$ is shown in Fig. 4 from which it is observed that the concentration decreases with the increase of the chemical reaction i.e the chemical reaction parameter is a decelerating agent and as a result, the solute boundary layer near the wall becomes thinner. This is due to fact that the conversion of the species takes place as a result of chemical reaction and thereby reduces the concentration in the boundary layer and hence increases the mass transfer. The effect of variable thermal conductivity parameter $\beta$ on temperature profiles is shown in Fig. 5. It is observed from the plot that increasing the value of $\beta$ results in increasing the magnitude of temperature due to increase in the thermal boundary layer thickness. Fig. 6 depicts the effect of the Schmidt number on the concentration profile. As the Schmidt number Sc increases, the mass transfer rate increases. Actually the Schmidt number is inversely proportional to the diffusion coefficient $D_m$. Hence, the concentration decreases with increasing Sc.The effect of transverse magnetic field on the velocity field, heat transfer and concentration profiles are depicted in Figs. 7a, 7b and 7c respectively. In Fig. 7a, it is observed that the velocity decreases with $\eta$ as the values of M increased. Thus, the presence of the magnetic field reduces the momentum boundary layer thickness and increases the power needed to stretch the sheet. The presence of moderate magnetic field can be used to stabilize the flow thereby delaying the transition from laminar to turbulent. Physically, the presence of transverse magnetic field gives rise to a drag force known as Lorentz force which results in retarding the velocity field. From the plots in Fig. 7b, it is observed that the transverse magnetic field contributes to the thickening of the thermal boundary layer. This is evident from the fact that the applied transverse magnetic field produces a body force, to be precise the Lorentz force, which opposes the motion. The resistance offered to the flow is responsible in enhancing the temperature. In Fig. 7c, it is observed that the concentration profile decreases with the increase of magnetic parameter $M$. Observation is very analogous with the theory because due to the transverse magnetic field a drag force is developed which opposes the flow.The influences of Eckert number on the dimensionless temperature function is shown in Fig. 8. The Eckert number designates the ratio of the kinetic energy of the flow to the boundary layer enthalpy differences. It embodies the conversion of the kinetic energy into internal energy by work done against the viscous fluid stresses. The positive Eckert number implies cooling of the plate i.e loss of heat from the plate to the fluid. Hence, greater viscous dissipative heat causes a rise in the temperature, which is evident from Fig. 8. Figs. 9a-9c displays effects of the porosity parameter on the velocity, temperature and concentration distributions. As shown in the figures, the temperature and concentration are increasing with increasing the dimensionless porosity parameter k and the velocity decreases as k increases. Physically, increasing the tightness of the porous medium which is represented by increase in k results in increasing the resistance against the flow and thus the fluid velocity decreased. The effects of velocity suction parameter $f_w$ on the velocity, temperature and concentration are shown in Figs. 10a, 10b and 10c respectively. It is found that an increase in suction parameter results a decrease in the velocity, temperature and concentration.

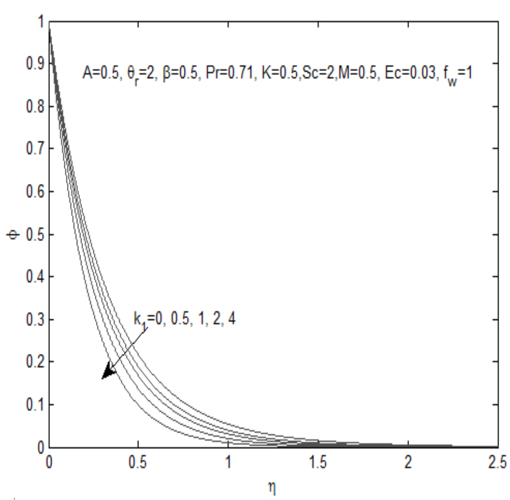


**Fig. 4. Concentration profiles for different values of chemical reaction rate $K_1$**

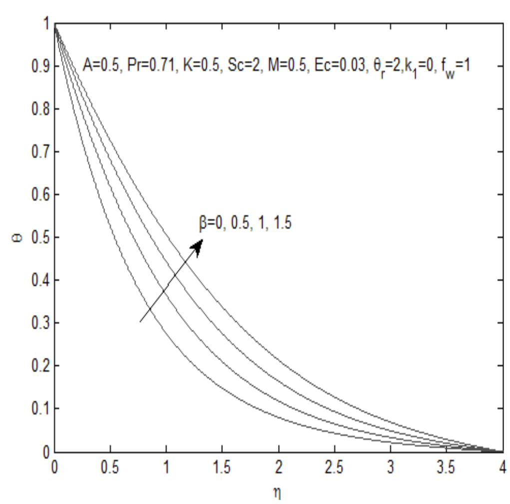


**Fig. 5. Temperature profiles for different values of variable thermal conductivity β**

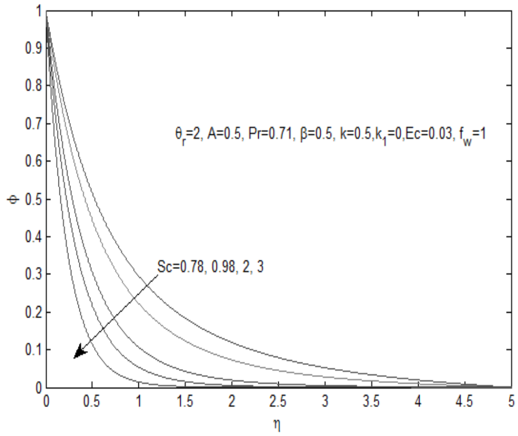


**Fig. 6. Concentration profiles for different values of Schmidt Sc**

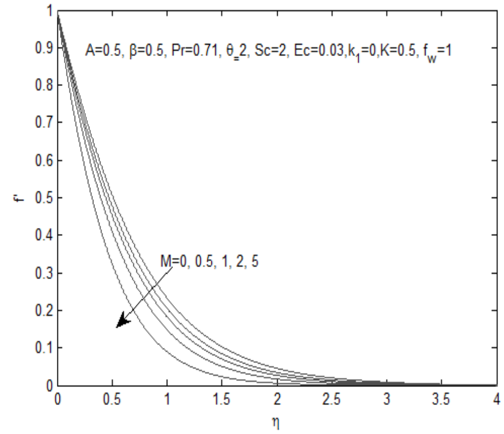


**Fig. 7a. Velocity profiles for different values of Magnetic parameter M**

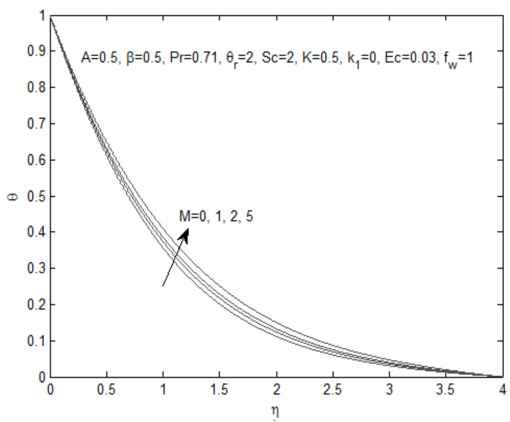


**Fig. 7b. Temperature profiles for different values of magnetic parameter M**

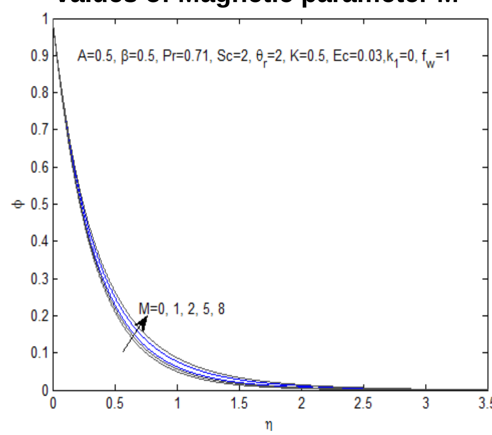


**Fig. 7c. Concentration profiles for different values of magnetic parameter M**

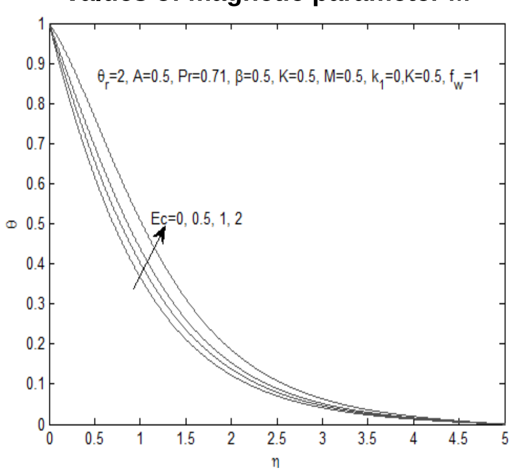


**Fig. 8. Temperature profiles for different values of Eckert number Ec**

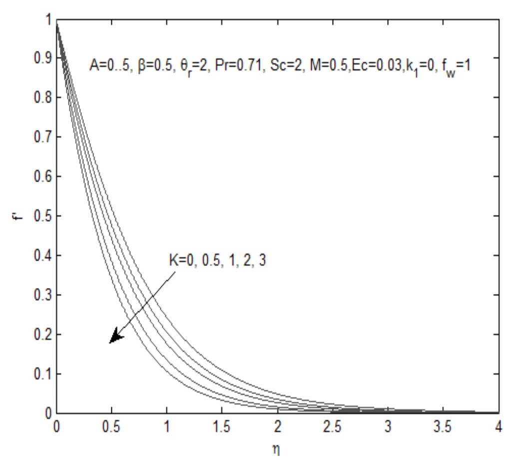


**Fig. 9a. Velocity profiles for different values of porous media parameter K**

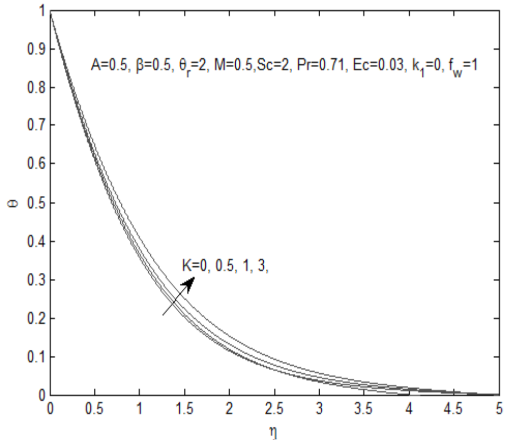


**Fig. 9b.Temperature profiles for different values of porous media parameter K**

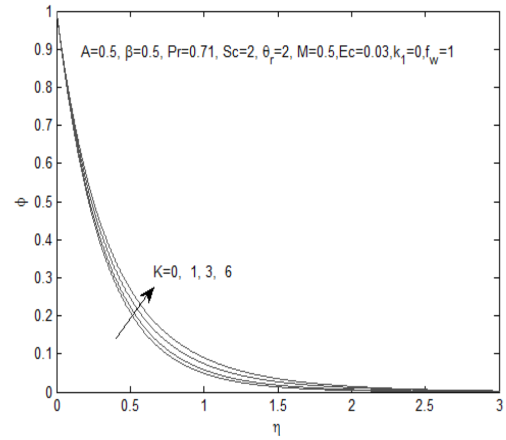


**Fig. 9c. Concentration profiles for different value of porous media parameter K**

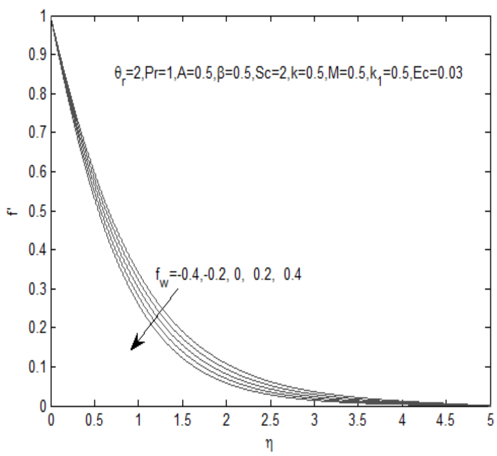


**Fig. 10a. Velocity profiles for different values of suction parameter $f_w$**

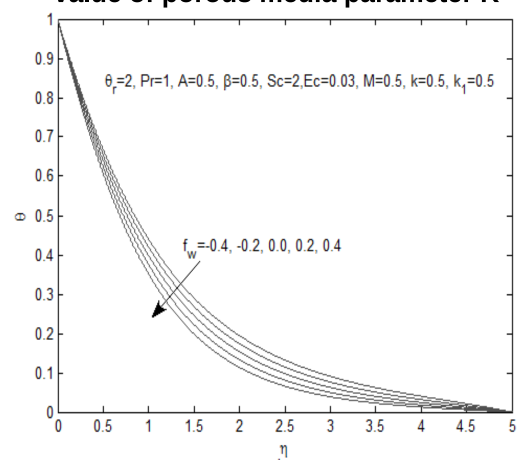


**Fig.10b.Temperature profiles for different values of suction parameter $f_w$**

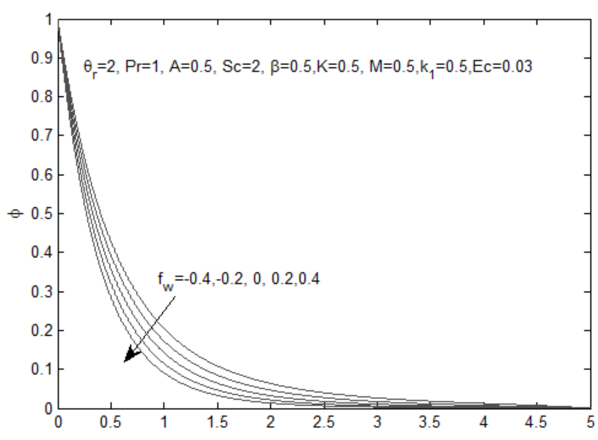


**Fig. 10c. Concentration profiles for different values of velocity suction parameter $f_w$**

## 5. CONCLUSION

In this work, an unsteady magnetohydrodynamic heat and mass transfer flow over stretching sheet embedded in porous medium with variable viscosity and thermal conductivity in presence of viscous dissipation and chemical reaction are investigated. The viscosity of the fluid is assumed to be an inverse linear function of temperature and the thermal conductivity is assumed to vary linearly with temperature. The resulting partial differential equations, which describe the problem, are transformed in to ordinary differential equations by using similarity transformations and then solved by numerically by fourth order Runge-Kutta method along with shooting technique. Velocity, temperature and concentration profiles are presented graphically and analyzed. The fundamental parameters found to affect the problem under consideration are variable viscosity, porous medium parameter, unsteadiness parameter, magnetic parameter, thermal conductivity parameter, chemical reaction parameter, Eckert number, Schmidt number and the velocity suction parameter. The findings of the numerical results can be summarized as follows:

- i. It is found that, the temperature as well as concentration increases with the increase of variable viscosity parameter, magnetic parameter and porosity parameter.
- ii. An increase in suction parameter/injection results a decrease in the velocity, temperature and concentration.
- iii. The concentration decreases with the increase of chemical reaction and Schmidt parameter but increases with the increase of unsteadiness parameter.
- iv. The temperature decreases with the increase of unsteadiness parameter and Prandtl number but increases with the increase of Eckert number as well as thermal conductivity parameter.
- v. It is found that an increase in suction parameter results a decrease in the velocity, temperature and concentration.
- vi. It is also found that the velocity decrease with the increase of viscosity, porous medium parameter, unsteadiness parameter, magnetic parameter, thermal conductivity parameter, chemical reaction parameter, Eckert number parameter and Schmidt number.

## COMPETING INTERESTS

Authors have declared that no competing interests exist.

## REFERENCES

1. Sakkiadis BC. Boundary layer behaviors on continuous surface. AlChE. 1961;7(2):221-225.
2. Erikson LE, Fan LT., Fox VG. Heat and mass transfer on a moving continuous plate with suction and injection. Ind. Eng. Chem. Fundamental.1966;5:19-25.
3. Tsou FK , Sparrow FM, Golldstien RJ. Flow and heat transfer in the boundary layer in continuous moving surface. Int. J. Heat Mass transfer. 1967;10:219-235.
4. Ellahi R, Hameed M. Numerical analysis of steady flows with heat transfer, MHD and nonlinear slip effects. International Journal for Numerical Methods for Heat and Fluid Flow. 2012;22(1):24-38.
5. Nadeem S, Rizwan H, Khan ZH. Numerical study of MHD boundary layer flow of a Maxwell fluid past a stretching sheet in the presence of nanoparticles. Journal of the Taiwan Institute of Chemical Engineers. 2014;45(1):121-126.

6. Nadeem S, Rizwan H, Noreen SA, Khan ZH. MHD three dimensional Casson fluid towards past a porous linearly stretching sheet. Alexandria Engineering Journal. 2013;52(4):577-582.
7. Chakrabarti A, Gupta AS. Hydromagnetic flow and heat transfer over stretching sheet, Quarterly Journal of Mechanics and Applied Mathematics. 1979;37:73-78.
8. Abdul R, Nadeem S. Heat transfer analysis of the boundary layer flow over a vertical exponential stretching cylinder. Global journal of Science Frontier Research mathematics and Decision Sciences. 2013;13(1):73-85.
9. Abdul R, Nadeem S. Mixed convection heat transfer in micropolarnanofluid over a vertical slender cylinder. Chinse physics letters. 2012;29(12):124701-124705.
10. Akbar NS, Nadeem S. Numerical solutions of magnetohydrodynamic boundary layer flow of tangent hyperbolic fluid towards a stretching sheet. Indian Journal of Physics. 2013;87(11):1121-1124.
11. Seddeek MA, Darwish AA, Abdelmeguid MS. 2007 Effects of chemical reaction and variable viscosity on hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through porous media with radiation. Comm. Non-Linear Sci. Numer. Simulat. 2007;12:195-213.
12. Ghay AY, Seddek MA. Chebyshev finite difference method for the effects of chemical reaction. Heat and mass transfer on laminar flow along a semi-infinite horizontal plate with temperature dependent viscosity. Chaos Solitons Fractas. 2004;19:61-70.
13. Zefar HK, Rahim G, Waqar AK. Effect of variable thermal conductivity on heat transfer from a hollow sphere with heat generation using homotopy perturbation method. ASME Heat Transfer Summer Confernce. 2008;301-309.
14. Rahman MM, Salahuddin KM. Study of hydromagnetic heat and mass transfer flow over an inclined heated surface with variable viscosity and electric conductivity. Comm. Non-linear Sci. Numer. Simulat. 2010;15(5):2073-2085.
15. Gupta PS, Gupta AS. Heat and mass transfer on a stretching with suction or blowing. Canad. J. chem. Engng. 1977;55:744-746.
16. Seddeek MA, Salama FA. The effects of temperature dependent viscosity and thermal conductivity on unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Compt. Mater. Sci. 2007;40(2):186-192.
17. Ambreen AK, Ellahi R, Usman M. Effects of variable viscosity on the flow of non-Newtonian fluid through a porous medium in an inclined channel with slip conditions. Journal of Porous Media. 2013;16(1):59-67.
18. Salem AM. Variable viscosity and thermal conductivity effects on MHD flow and heat transfer in isco-elastic fluid over a stretching sheet. Phys. Lett. 2007;369:315-322.
19. Hassan M, Soleiman S. Effects of MHD and temperature dependent viscosity on the flow of non-Newtonian nanofluid in pipe: Analytical solutions. Applied Mathematical Modeling. 2013;37(3):1451-1457.
20. Hassanian LA. Alarabi TH. Non-Darcy unsteady mixed convection flow near the stagnation point on a heated vertical surface embedded in a porous medium with thermal radiation and variable viscosity. Commun. Nonlinear Sci. Numer. Simulat. 2009;14:1366-1376.
21. Mohammed RH, Nourazar S. Conjugated forced convection heat transfer from a heated flate plate of finite thickness and temperature dependent thermal conductivity. Heat Transfer Engineering. 2014;35:863-874.
22. Gehart B, Pera L. The nature of vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass diffusion. International Journal of Heat Mass Transfer. 1971;14:2025-2050.

23. Byron Bird R, Warren E Stewart, Edwin N. Lightfoot. Transport phenomena. John Wiley and Sons, New York; 1992.
24. Cussler EL. Diffusion Mass Transfer in Fluid Systems, Cambridge University Press, London, UK; 1988.
25. Shakhaoath MK, Ifsana K, Sirajul MI. Possessions of chemical reaction on MHD heat and mass transfer of nanofluid flow on a continuously moving surface. American Chemical Science Journal. 2014;4(3):401-415.
26. Apelblat A. Mass transfer with a chemical reaction of the first order. Effects of axial diffusion. The Chemical Engineering Journal. 1982;23:193-203.
27. Das UN, Deka R, Soundalgekar. Effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Forschung im Ingenieurwesen. 1994;60:284-87.
28. Muthucumaraswamy R, Ganesan P. Effects of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate. Journal of Applied Mechanics and Technical. Physics. 2001;42:665-671.
29. Mahajan RL, Gebhart BB. Viscous dissipation effects in buoyancy induced flows. Int. J. Heat Mass transfer. 1989;32(7):1380-1382.
30. Mohammad RH, Nourazar SS, Campo A. Analytical solution for two-phase flow between two rotating cylinders filled with power law liquid and a micro layer of gas.Journal of Mechanical Science and Technology. 2014;28(5):1849-1854. DOI:10.1007/s12206-013-0913-y.
31. Gebharat B, Mollendorf J. Viscous dissipation in external natural convection flows. J. Fluid. Mech. 1969;38:97-110.
32. Soundalgekar VM. 1972 viscous dissipation effects on unsteady free convective flow past an infinite, vertical porous plate with constant suction. Int. J. Heat Mass Transfer. 1972;15:1253-1261.
33. Israel-Cookey, Ogulu A, Omubo-Pepple VB. 2003 Influence of viscous dissipation on unsteady MHD free convection flow past an infinite heated vertical plate in porous medium with time-dependent suction. Int. J. heat mass transfer. 2003;46:2305-2311.
34. Goverdhan K, Kishan N, Balaswamy. Effect of viscous dissipation and radiation on MHD gas flow and heat and mass transfer over a stretching sheet surface with a uniform free stream. Journal of Engineering physics and Thermophysics. 2012;85(4):909-916.
35. Ramachandra Prasad V, Bhaskar Reddy N. 2007 Radiation and mass transfer effects on unsteady MHD free convection flow past a heated vertical plate in a porous medium with viscous dissipation.
36. Mohammad RH, Nourazar SS. Semi-analytical treatments of conjugate heat transfer. Journal of Mechanical Engineering Science. 2012;227:492-503.
37. Ali HM, Mohammad RH, Nourazar SS. On the solution of charactersic value problems arising in linear stability analysis; Semi-analytical approach. Applied Mathematics & Compputaion. 2014;239:126-132.
38. Abdou MMM. 2010. Effect of radiation with temperature dependent viscosity and thermal conductivity on unsteady a stretching sheet through porous media. Nonlinear Analysis: Modelling and control. 2010;15(3):257-270.
39. Ling JX, Dybbs A. Forced convection over a flat plate submersed in a porous medium: Variable viscosity case. ASME Winter Annual meeting, Boston. 1987;13-18.
40. Lai FC, Kulacki FA. The effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium. Int. J. Hear Mass transfer. 1990;33:1028-1031.

41. Hunsain S, Mehmood A, Ali A. 2012 Heat and mass transfer analiyisis in unsteady boundary layer flow through porous media with variable viscosity and thermal diffusivity. Journal of Appl. Mechanics and Tech. physics. 2012;53(5):722-723.
42. Na TY. Computational methods in engineering boundary values. Academic Press, New York; 1979.



*Peer-review history:*
*The peer review history for this paper can be accessed here:*
*http://www.sciencedomain.org/review-history.php?iid=528&id=16&aid=5457*