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Effect of Centrifugal Force on a Porous Anisotropic Medium in Rotation, Saturated by a Non-Newtonian Fluid

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Authors' contributions

This work was done in collaboration among all authors. Author VEC designed the study, carried out the statistical analysis, drafted the protocol and drafted the first draft of the manuscript. Authors AC and SGC managed analyzes of the study, judged the relevance of the study carried out and verified all the established equations. Authors SAE and DG managed the documentary research. All authors read and approved the final manuscript.

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ABSTRACT

The present study deals with the linear stability of an anisotropic porous medium in rotation, saturated by a non-Newtonian fluid in a rectangular cavity heated on the side, subjected to the effect of the centrifugal force. The state of marginal stability is established by determining the critical Rayleigh number and the critical wave number. We have observed the effect of the parameters K^* and φ of the anisotropy on the convection threshold.

Keywords: Rotating anisotropic porous media; centrifugal force; linear stability; critical Rayleigh number; convection threshold.

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NOMENCLATURES

μ : Dynamic viscosity of the fluid saturating the porous medium
$\vec{g}$: Vector gravity field
p' : Pressure of the saturating vapor
$\vec{\Omega}\,(0,0,\Omega)$: Vector rotation speed of the enclosure
$\vec{r}'$: Vector position
$\overline{\overline{K}}$: Third order permeability tensor
σ : Thermal capacity ratio
$\vec{V}$: Vector velocity of filtration of the fluid in the porous medium;
$\Delta T = T_H - T_C$: Temperature difference between the two surfaces;
$R_a = \frac{K_1 \rho_o \Omega^2 H^2 \beta \Delta T}{\mu \alpha}$: Number of Rayleigh for the centrifugal force;
u'; v' and w': Components of the velocity vector respectively along the axes $(o,x),(o,y)$ and (o,z)
k_y and k_z: Wave numbers describing the periodicity of the disturbance in the directions y and z respectively
${k_y}^2 + {k_z}^2 = k^2$: Wave number
$\delta_{mn} = \begin{cases} 1\ when\ \ m = n \\ 0\ when\ \ m \neq n \end{cases}$ and $\delta_{m+n,2p-1} = \begin{cases} 1\ when\ \ m+n\ \ it\ is\ odd\ nomber \\ 0\ when\ \ m+n\ \ it\ is\ even\ nomber \end{cases}$ the Kronecker
$a = cos^2\varphi + K^* sin^2\varphi$; $b = sin^2\varphi + K^* cos^2\varphi$ and $c = \frac{1}{2}(K^* - 1)\, sin2\varphi$: Elements of the inverse of the permeability tensor
φ : Angle of orientation of the main directions of the permeability tensor
k_1, k_2 and k_3: Permeabilities along the main directions
$K^* = K_1/K_2$ and $\zeta = K_1/K_3$: Permeability anisotropy ratio
ρ: Density of the fluid
$\hat{e}_x$; $\hat{e}_y$ and $\hat{e}_z$: Unitary vectors in the main directions

1. INTRODUCTION

The present paper deals with the study of linear stability in a rotating anisotropic porous medium saturated by a viscoelastic fluid, heated from below. The study of viscoelastic fluids is of great interest in many fields of modern engineering science and technology such as materials processing, petroleum, chemical and nuclear industries, geophysics, biology and of bio-mechanical engineering [1,2,3].

Vadasz [4] carried out three-dimensional analytical research on the flow of a fluid through a heterogeneous porous medium confined in a rectangular cavity $0 \leq x \leq L$; $0 \leq y \leq H$; $0 \leq z \leq H$ in rotation. The permeability of the porous medium varies in the z coordinate direction. The results of his work show that for a pressure gradient applied to the faces $x = 0$ and $x = L$, a main flow of the fluid appears in the horizontal axis direction $(o, \overrightarrow{x})$. The, analytical solution found remains valid for large numbers of Ekman (Ek), which confirms the conditions of practical applications.

The same author has numerically investigated the effects of the centrifugal force resulting from the rotation of a rectangular cavity on the phenomenon of thermal convection in the porous medium confined in this chamber. He showed that for simple analytical solutions, the geometric shape ratio where H is the height of the enclosure and L the length of the rectangular domain, greatly influences the flow and heat transfer depends on the number of modified Rayleigh Ra_w which must remain low. He established that for the two-dimensional flow in the porous medium, the Nusselt number depends linearly on the number of modified Rayleigh Ra_w with a slope function of the number " a " (geometrical shape factor of the domain). The author has also investigated an analytical solution to the problem of natural convection generated by centrifugal force in the porous medium contained in this rotating field and heated from above. For assuming the vertical component ω of the flow velocity and temperature T independent of the horizontal coordinate x, $W(x,z) = W(z)$ and $T(x,z) = T(z)$, it found that the range of validity of this analytical solution must be limited. The number of Nusselt varies linearly with the number of Rayleigh changes Ra_w for low values of the latter. It emerges that apart from the heat flow associated with the flow of the fluid in the x coordinate axis direction which remains large, a heat flow in the vertical direction occurs.

Vadasz and Saneshan [5] carried out a numerical and analytical investigation on the two-dimensional natural convection in isotropic porous medium in permeability confined in a rotating rectangular enclosure. The effect of centrifugal force on the development and stability of natural convection in a vertical porous layer exposed laterally to a constant flow of heat collinear to centrifugal force has been investigated [6,7].

Enock and Tyvand [8] on the basis of the Darcy-Boussinesq equations for thermal convection in a rotating porous medium, studied the steady-state problem of two-dimensional convection in a rotating porous layer. They showed that this problem is equivalent to that of the porous anisotropic medium relative to a new dimensionless variable ζ, characterizing the permeability ratio of the medium. According to the results they obtained, one can deduce basic results on thermal convection in a porous layer in rotation from the analysis made on the thermal convection in anisotropic porous medium not put in rotation movement.

Jong and Jian [9] studied transient thermal convection in a porous layer whose free upper and lower surfaces, initially at the same temperatureT_o, are subjected to constant heating from below.

Jong and Jian [9] have also studied analytically and numerically the criteria for the appearance in steady state and transient of the two-dimensional thermal convection in a rotating porous medium. The porous permeable anisotropic medium is such that its upper and lower boundary boundaries are rigid. The lower rigid wall is heated at a constant rate of heating so as to generate a linear distribution of the temperature in the vertical direction [10,1]. The instability related to the anisotropy permeability of porous regime, saturated with a fluid was analyzed by the technique of calculating the average flow volume [11,12]. They determined the critical Rayleigh numbers R_c and critical wave numbers a_c for the appearance of convection in the anisotropic medium.

Govender [13] studied natural convection in an anisotropic porous spinning layer subjected to centrifugal force. He used Darcy's equation to describe the flow and found that convection is stabilized when the ratio of anisotropic thermal and mechanical parameters increases in amplitude.

Nield and Bejan [14] have established comprehensive reviews of the fundamental principles of heat convection in porous media.

Dègan [15,16] conducted a numerical and analytical investigation of natural convection in a rectangular cavity confined by a porous permeable anisotropic medium and isothermally heated by the sides. The main axes of the permeability are chosen inclined at an angle φ with respect to the gravitational field. The results showed that the permeability anisotropy ratio $K^*$and the angle of inclination φ of the principal axes both have a great influence on the system. In particular, the maximum (minimum) heat transfer is obtained when the orientation of the main axis of the porous anisotropic medium having the permeability is parallel (perpendicular) to the gravitational field.

2. MATERIALS AND METHODS

2.1 Description of the Physical Model

The physical model considered in Fig. 1 is that of a parallelepipedic enclosure with flat walls. The lower horizontal wall is that symbolizing the pan which is heated by one of the side faces at a constant temperature T_H while the other side face is at constant temperature T_C , such that $(T_H > T_C)$.

The cassava paste contained in the chamber constitutes a porous medium saturated with starch assimilable to a non-Newtonian fluid. The porous medium is anisotropic permeability whose directions are oriented obliquely to the vertical axis (Fig. 1). The porous-pregnant medium system is subjected to a maintained rotational movement of constant frequency N.

As soon as the heating begins, the porous anisotropic medium is the site of thermo-convective phenomena that we will study.

2.2 Governance Equations

The equations governing our system are written:

$$\vec{\nabla}.\vec{V'} = 0 \quad (1)$$

$$\vec{V'} = \frac{\bar{\bar{K}}}{\mu}\left[-\vec{\nabla}P' + \frac{1}{2}\rho\vec{\nabla}\left|\vec{\Omega}\wedge\vec{r'}\right|^2\right] \quad (2)$$

$$\sigma\frac{\partial T'}{\partial t'} + \vec{V'}.\left(\vec{\nabla}T'\right) = \alpha\nabla^2 T' \quad (3)$$

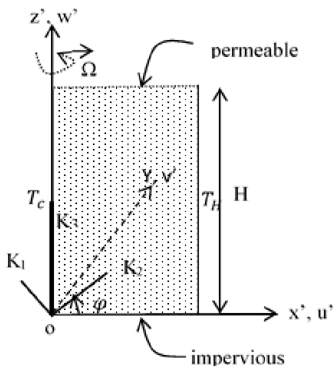

Fig. 1. Rectangular rotating cavity and coordinate axes, containing the anisotropic porous medium

2.2.1 Dimensionalization of the equations

From the normalization scale, we have the following dimensionless variables:

By introducing the following adimensioned variables:

$$x=\frac{x^{'}}{H};\ y=\frac{y^{'}}{H};\ u=\frac{u^{'}}{\alpha/_H};v=\frac{v^{'}}{\alpha/_H};$$
$$w=\frac{w^{'}}{\alpha/_H};\ T=\frac{T^{'}-T_o}{\Delta T};\ t=\frac{t^{'}}{H^2\,\sigma/\alpha};$$
$$p=\frac{K_1}{\alpha\mu}P^{'} \tag{4}$$

in equations (1), (2) and (3), we obtain the following:

$$\vec{\nabla}.\vec{V}=0 \tag{5}$$

$$\left(\overline{\overline{K}}\right)^{-1}\vec{V}=-\vec{\nabla}P-R_a x T\vec{\imath} \tag{6}$$

$$\frac{\partial T}{\partial t}+\vec{\nabla}.\left(\vec{V}T\right)=\nabla^2\mathrm{T} \tag{7}$$

Whère $R_a=\frac{K_1\rho_o\Omega^2H^2\beta\Delta T}{\mu\alpha}$: Rayleigh number for centrifugal force.

2.2.2 Equations in the disturbed state

In the basic state, we have pure conduction:

$$u_b\ =v_b\ =w_b=0;\qquad P_b=-\frac{1}{3}R_a x^3+cste;\qquad T_b=x$$

The question is to know if this solution without movement of the fluid will always prevail, whatever the difference of temperature ΔT that we will impose. We will answer this question by launching a stability experiment of the type described in the framework of the convection transition of laminar-turbulent flow. The linear stability experiment consists of disturbing the basic solution and observing under which conditions the imposed perturbation increases in amplitude. So, we substitute

$$\begin{aligned}
T(x,y,z,t) &= T_b(z) + T'(x',y',z',t')\\
v(x,y,z,t) &= 0 + v'(x',y',z',t')\\
u(x,y,z,t) &= 0 + u'(x',y',z',t')\\
w(x,y,z,t) &= 0 + w'(x',y',z',t')\\
\underbrace{p(x,y,z)}_{\text{Transient}} &= \underbrace{p_{b(z)}}_{\text{Basic}} + \underbrace{p'(x',y',z')}_{\text{Disturbance}}
\end{aligned}$$

in the equations governing the transitional regime; we obtain for the following equations in the disturbed state:

$$\vec{\nabla}.\vec{V'}=0 \tag{9}$$

$$\left(\overline{\overline{K}}\right)^{-1}\vec{V}'=-\vec{\nabla}P'-R_a xT'\hat{e}_x \tag{10}$$

$$\left(\frac{\partial}{\partial t}-\nabla^2\right)T'+u'\frac{\partial T'}{\partial x}+v'\frac{\partial T'}{\partial y}+w'\frac{\partial T'}{\partial z}+u'=0 \tag{11}$$

in equation (11), we have the possibility of eliminating nonlinear terms $(u^{'}\frac{\partial \boldsymbol{T}^{'}}{\partial x^{'}};\boldsymbol{v}^{'}\frac{\partial \boldsymbol{T}^{'}}{\partial y^{'}}\ \boldsymbol{et}\ w^{'}\frac{\partial \boldsymbol{T}^{'}}{\partial z^{'}})$ assuming that the fluid flow rate and the disturbance temperature are negligible. Thus, in equation (11), only the terms of the first order are retained. So we get:

$$\left(\frac{\partial}{\partial t}-\nabla^2\right)T^{'}+u^{'}=0 \tag{12}$$

We will eliminate the pressure terms in equation (10) by taking twice the nabla ($\vec{\nabla}$) vector equation (10), we obtain the system of equations (13) below:

$$\begin{cases}
\left(\zeta\frac{\partial^2 \boldsymbol{w'}}{\partial \boldsymbol{x}\partial \boldsymbol{z}}-\boldsymbol{c}\frac{\partial^2 \boldsymbol{u'}}{\partial \boldsymbol{y}\partial \boldsymbol{z}}+\boldsymbol{b}\frac{\partial^2 \boldsymbol{v}^{'}}{\partial \boldsymbol{y}\partial \boldsymbol{x}}-\boldsymbol{a}\frac{\partial^2 \boldsymbol{u'}}{\partial \boldsymbol{y}^2}-\boldsymbol{a}\frac{\partial^2 \boldsymbol{u'}}{\partial \boldsymbol{z}^2}+\boldsymbol{c}\frac{\partial^2 \boldsymbol{v'}}{\partial \boldsymbol{y}^2}+\boldsymbol{c}\frac{\partial^2 \boldsymbol{v}^{'}}{\partial \boldsymbol{z}^2}\right)=Ra.x\left(\frac{\partial^2 \boldsymbol{T}^{'}}{\partial \boldsymbol{y}^2}+\frac{\partial^2 \boldsymbol{T}^{'}}{\partial \boldsymbol{z}^2}\right)\\
-\left(-\zeta\frac{\partial^2 \boldsymbol{w}^{'}}{\partial \boldsymbol{x}^2}-\boldsymbol{c}\frac{\partial^2 \boldsymbol{u}^{'}}{\partial \boldsymbol{x}^2}+\boldsymbol{b}\frac{\partial^2 \boldsymbol{v}^{'}}{\partial \boldsymbol{x}^2}-\boldsymbol{a}\frac{\partial^2 \boldsymbol{u}^{'}}{\partial \boldsymbol{x}\partial \boldsymbol{y}}-\boldsymbol{c}\frac{\partial^2 \boldsymbol{u}^{'}}{\partial \boldsymbol{z}^2}+\boldsymbol{c}\frac{\partial^2 \boldsymbol{v}^{'}}{\partial \boldsymbol{x}\partial \boldsymbol{y}}+\boldsymbol{b}\frac{\partial^2 \boldsymbol{v}^{'}}{\partial \boldsymbol{z}^2}\right)=-Ra\left(\boldsymbol{x}\frac{\partial^2 \boldsymbol{T}^{'}}{\partial \boldsymbol{x}\partial \boldsymbol{y}}+\frac{\partial \boldsymbol{T}^{'}}{\partial \boldsymbol{y}}\right)\\
\left(-\zeta\frac{\partial^2 \boldsymbol{w'}}{\partial \boldsymbol{y}\partial \boldsymbol{z}}+\boldsymbol{a}\frac{\partial^2 \boldsymbol{u'}}{\partial \boldsymbol{y}\partial \boldsymbol{z}}-\boldsymbol{c}\frac{\partial^2 \boldsymbol{v'}}{\partial \boldsymbol{x}\partial \boldsymbol{z}}-\boldsymbol{a}\frac{\partial^2 \boldsymbol{u'}}{\partial \boldsymbol{x}\partial \boldsymbol{y}}-\xi\frac{\partial^2 \boldsymbol{w'}}{\partial \boldsymbol{y}^2}+\boldsymbol{c}\frac{\partial^2 \boldsymbol{u'}}{\partial \boldsymbol{y}\partial \boldsymbol{z}}-\boldsymbol{b}\frac{\partial^2 \boldsymbol{v'}}{\partial \boldsymbol{y}\partial \boldsymbol{z}}\right)=-Ra\left(\boldsymbol{x}\frac{\partial^2 \boldsymbol{T}^{'}}{\partial \boldsymbol{x}\partial \boldsymbol{z}}+\frac{\partial \boldsymbol{T}^{'}}{\partial \boldsymbol{z}}\right)
\end{cases} \tag{13}$$

In this system of equations, the component along the axis (o, x) suffices to solve the problem; (according to Vadasz [4]); so we get:

$$\begin{cases} \frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}+\frac{\partial w'}{\partial z}=0 \\ \left(\zeta\frac{\partial^2 w'}{\partial x\partial z}-c\frac{\partial^2 u'}{\partial y\partial z}+b\frac{\partial^2 v'}{\partial y\partial x}-a\frac{\partial^2 u'}{\partial y^2}-a\frac{\partial^2 u'}{\partial z^2}+c\frac{\partial^2 v'}{\partial y^2}+c\frac{\partial^2 v'}{\partial z^2}\right)=Ra.x\left(\frac{\partial^2 T'}{\partial y^2}+\frac{\partial^2 T'}{\partial z^2}\right) \\ \left(\frac{\partial}{\partial t}-\nabla^2\right)T'+u'=0 \end{cases} \tag{14}$$

We will rewrite these equations assuming that $\vec{V}'(u',0,w')$ and we will use the continuity equation$\left(\frac{\partial^2 \boldsymbol{w}'}{\partial x\partial z}=-\frac{\partial^2 \boldsymbol{u}'}{\partial x^2}\right)$. The system of equations to be solved thus becomes:

$$\begin{cases} \left[a\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)+\zeta\frac{\partial^2}{\partial x^2}+c\frac{\partial^2}{\partial y\partial z}\right]u'+ \\ \quad Ra.x\left(\frac{\partial^2 T'}{\partial y^2}+\frac{\partial^2 T'}{\partial z^2}\right)=0 \\ \quad \left(\frac{\partial}{\partial t}-\nabla^2\right)T'=-u' \end{cases} \tag{15}$$

Applying the operator $\left(\frac{\partial}{\partial \boldsymbol{t}}-\nabla^2\right)$ to the first line of this system of equations (15), we obtain the temperature perturbation equation in the form:

$$\left\{\begin{matrix} \left(\frac{\partial}{\partial \boldsymbol{t}}-\nabla^2\right)\left[\boldsymbol{a}\left(\frac{\partial^2}{\partial \boldsymbol{y}^2}+\frac{\partial^2}{\partial \boldsymbol{z}^2}\right)+\boldsymbol{\zeta}\frac{\partial^2}{\partial \boldsymbol{x}^2}+\boldsymbol{c}\frac{\partial^2}{\partial \boldsymbol{y}\partial \boldsymbol{z}}\right]- \\ Ra.x\left(\frac{\partial^2}{\partial \boldsymbol{y}^2}+\frac{\partial^2}{\partial \boldsymbol{z}^2}\right) \end{matrix}\right\}\boldsymbol{T}'=0 \tag{16}$$

2.2.3 Analysis in normal mode

The initial condition for this transitory problem is arbitrary; but, inspired by the visual observations of Benard's cells, it is logical to assume sinusoidal variables in (y and z) and the exponential variable in t:

$$T=A_n\Theta(x)e^{\sigma t+i(k_y y+k_z z)} \tag{17}$$

By substituting equation (17) in equation (16), we obtain:

$$\left[(D^2-k^2-\sigma)(\zeta D^2-ak^2)+ick_y(D^2-k^2-\sigma)D-R_a x k^2\right]\Theta=0 \tag{18}$$

Where $D\equiv\frac{d}{dx}$

The appropriate boundary conditions are:

$$\begin{cases} x=0, \ \ \Theta=0 \\ x=1, \ \ \Theta=1 \end{cases}$$

Equation (18) will be solved by the Galerkin method. Therefore, $\Theta(x)$ is developed in sets of orthogonal trivial functions that satisfy the boundary conditions defined in equation (19):

$$\Theta(x) = \sum_{i=1}^{N} a_n\, sin(n\pi x) \quad (19)$$

By introducing the expression of Θ (x) into equation (18), we obtain:

$$\sum_{n=1}^{N} a_n \{[(n^2\pi^2 + k^2 - \sigma)(\zeta n^2\pi^2 + ak^2) - R_a k^2 x]\sin(n\pi x) - ick_y n\pi(n^2\pi^2 + k^2 - \sigma)\cos(n\pi x)\} = 0 \quad (20)$$

With

$i = \sqrt{-1}$, k_y and k_z the wave numbers describing the periodicity of the perturbation in the directions y and z respectively with ${k_y}^2 + {k_z}^2 = k^2$. σ being the rate of increase of the stability.

2.2.4 Study of marginal stability: $\sigma = 0$

We will multiply the equation (20) by $\sin(m\pi x)$ and integrate on the domain [0; 1]. So, we get:

$$\sum_{n=1}^{N} a_n \left\{ [(n^2\pi^2 + k^2)(\zeta n^2\pi^2 + ak^2)] \int_0^1 \sin(m\pi x)\sin(n\pi x)\,dx - R_a k^2 \int_0^1 x\sin(m\pi x)\sin(n\pi x)dx \right.$$
$$\left. - ick_y n\pi(n^2\pi^2 + k^2)\int_0^1 \sin(m\pi x)\cos(n\pi x)dx \right\} = 0 \quad (21)$$

$$\sum_{n=1}^{N} a_n \left\{ \left[(n^2\pi^2 + k^2)(\zeta n^2\pi^2 + ak^2) - \frac{1}{2}R_a k^2\right]\frac{\delta_{mn}}{2} + \left[\frac{4m.n.R_a k^2}{\pi^2(m^2 - n^2)^2} - \frac{i4m.n.ck_y(n^2\pi^2 + k^2)}{\pi(m^2 - n^2)}\right]\delta_{m+n,2p-1} \right\} = 0 \quad (22)$$

with $\delta_{mn} = \begin{cases} 1\ when\ m = n \\ 0\ when\ m \neq n \end{cases}$ and $\delta_{m+n,2p-1} = \begin{cases} 1\ when\ m+n\ is\ odd\ nomber \\ 0\ when\ m+n\ is\ oven\ nomber \end{cases}$

N = 1,2 ...., 7 and P any integer

Set m = 1 and let n = 1 to 2 be varied, we get:

$$\frac{1}{2}\left[(\pi^2 + k^2)(\zeta\pi^2 + ak^2) - \frac{1}{2}R_a k^2\right]a_1 + \left[\frac{8R_a k^2}{9\pi^2} + \frac{i8ck_y(4\pi^2 + k^2)}{3\pi}\right]a_2 = 0 \quad (23)$$

Set m = 2 and vary n = 1 to 2, we get:

$$\left[\frac{8R_a k^2}{9\pi^2} - \frac{i8ck_y(\pi^2 + k^2)}{3\pi}\right] + \frac{1}{2}\left[(4\pi^2 + k^2)(4\pi^2\zeta + ak^2) - \frac{1}{2}R_a k^2\right]a_2 = 0 \quad (24)$$

Let us write the matrix form Lan = 0 of equations (23) and (24), we obtain:

$$\begin{bmatrix} \frac{1}{2}\left[(\pi^2 + k^2)(\zeta\pi^2 + ak^2) - \frac{1}{2}R_a k^2\right] & \left[\frac{8R_a k^2}{9\pi^2} + \frac{i8ck_y(4\pi^2 + k^2)}{3\pi}\right] \\ \left[\frac{8R_a k^2}{9\pi^2} - \frac{i8ck_y(\pi^2 + k^2)}{3\pi}\right] & \frac{1}{2}\left[(4\pi^2 + k^2)(4\pi^2\zeta + ak^2) - \frac{1}{2}R_a k^2\right] \end{bmatrix} \begin{bmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \end{bmatrix} = 0 \quad (25)$$

This equation has a solution if det (L) = 0⟺

$$\Rightarrow \left(\frac{1}{16} - \frac{64}{81\pi^4}\right)k^4 R_a^2 - \frac{k^2}{8}[(\pi^2 + k^2)(\zeta\pi^2 + ak^2) + (4\pi^2 + k^2)(4\pi^2\zeta + ak^2)]R_a + \frac{1}{4}(\pi^2 + k^2)(\zeta\pi^2 + ak^2)(4\pi^2 + k^2)(4\pi^2\zeta + ak^2)$$
$$- \frac{64{k_y}^2 c}{9\pi^2}(\pi^2 + k^2)(4\pi^2 + k^2)^2 - \frac{i64ck_y k^2}{9\pi}R_a = 0 \quad (26)$$

The physical meaning of the Rayleigh number is that this number is a real number; the imaginary part of equation (26) is therefore zero:

$$\Rightarrow \frac{6\ 4 c k_y k^2}{9\pi} = 0 \qquad (27)$$

with c and k are non-zero; so it's ky = 0

Equation (26) then becomes:

$$\left(\frac{1}{16} - \frac{6\ 4}{81\pi^4}\right) k^4 R_a^2 - \frac{k^2}{8} \mathcal{L} R_a + \frac{1}{4}\delta = 0 \qquad (28)$$

With
$$\begin{cases} \delta = (\pi^2 + k^2)(\zeta\pi^2 + ak^2)(4\pi^2 + k^2)(4\pi^2\zeta + ak^2) \\ L = [(\pi^2 + k^2)(\zeta\pi^2 + ak^2) + (4\pi^2 + k^2)(4\pi^2\zeta + ak^2)] \end{cases}$$

$$\Rightarrow \frac{(3\pi)^4 - 1024}{(3\pi)^4} k^4 R_a^2 - 2\mathcal{L} k^2 R_a + 4\delta = 0$$

$$\Delta' = (\mathcal{L}k^2)^2 - 4\delta\left(\frac{(3\pi)^4 - 1024}{(3\pi)^4} k^4\right) > 0$$

$$R_a = \frac{\pounds k^2 \pm \sqrt{(\mathcal{L}k^2)^2 - 4\delta\left(\frac{(3\pi)^4 - 1024}{(3\pi)^4} k^4\right)}}{\left(\frac{(3\pi)^4 - 1024}{(3\pi)^4} k^4\right)}$$

Let

$$\gamma = \frac{(3\pi)^4 - 2^{10}}{(3\pi)^4}$$

The expression of $\boldsymbol{R_a}$ becomes:

$$R_a = \frac{1}{\gamma k^2}\left(\mathcal{L} \pm \sqrt{\mathcal{L}^2 - 4\delta\gamma}\right) \qquad (29)$$

3. RESULTS AND DISCUSSION

Figs. 2, 3 and 4 respectively illustrate the effect of the permeability anisotropy ratio $K^* = K_1/K_2$, the orientation angle φ of the main axes of permeability and the effect of the anisotropy ratio. $\zeta = K_1/K_3$ on the linear stability of convection in a porous anisotropic medium saturated by a non-Newtonian fluid.

In Fig. 2, for $K^* = 1$ (isotropic medium), the value of the critical Rayleigh number $Ra_C = 77{,}0829$ for a critical wave number of $k_c = 3{,}19$.

These results correspond well to those found by Vadasz [4] $Ra_w = 77{,}0829, k_C = 1{,}017\pi$. We find that the critical Rayleigh number increases with K^*. In other words if K^* < 1, the critical Rayleigh number $Ra_C$is lower and if K^*> 1, this number is high. This shows that the ratio of permeability anisotropy in the principal directions K_1 and K_2 has an effect on the initiation of convection in the medium.

For Fig. 3, the critical Rayleigh number decreases with the axis orientation angle permeability.

Indeed, the beginning of the convection is quickly reached when the main directions of the axes of permeability deviate.

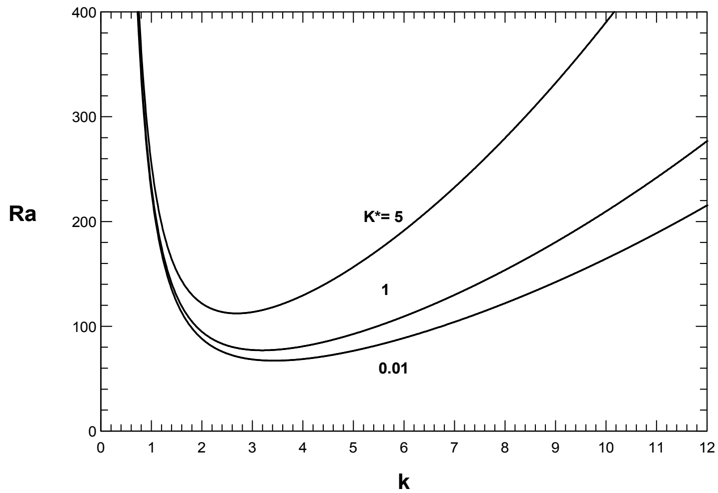

Fig. 2. Variation of Ra as a function of k for different values of $K^*$with $\zeta = 1$ and $\varphi = 45°$

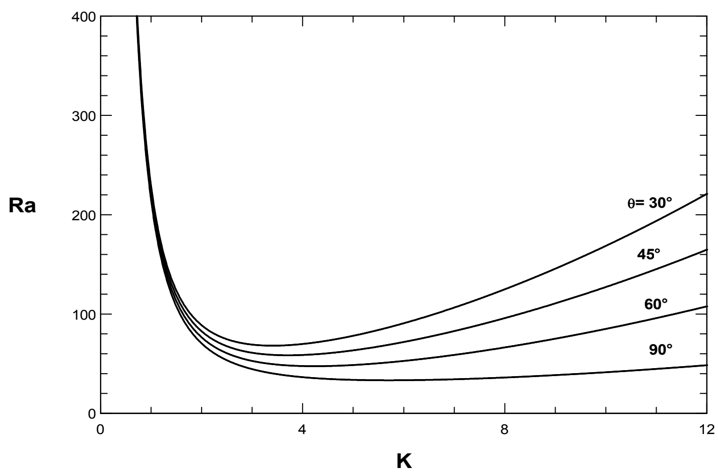

Fig. 3. Variation of Ra as a function of k for different values of φ with $K^* = 0.1$ and $\zeta = 1$

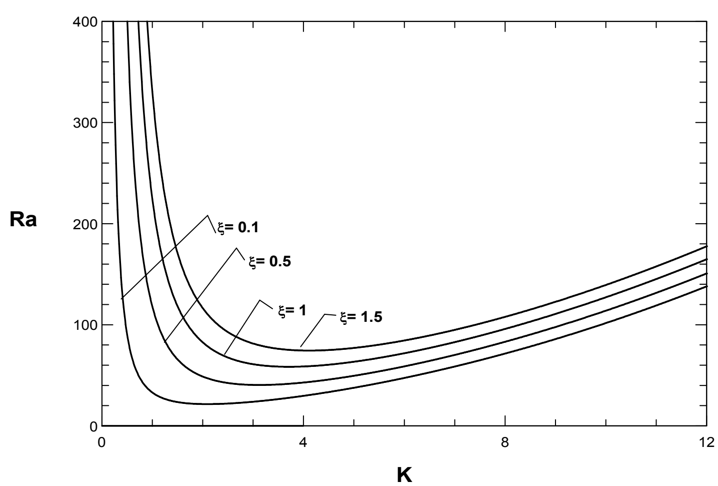

Fig. 4. Variation of Ra as a function of k for different values of ζ with $K^* = 0.1$ and $(\varphi = 45°)$

For Fig. 4, we find that the critical Rayleigh number increases with ζ.

In other words if ζ <1, the critical Rayleigh number $Ra_C$is lower and if ζ > 1, this number is high. This shows that the ratio of permeability anisotropy in the main directions $K_1$and K_3has an effect on the initiation of convection in the medium.

4. CONCLUSION

The problem of the linear stability of an anisotropic porous medium in rotation subjected to the action of a centrifugal force has been studied. From this study, the following conclusions emerge:

- A low value of the permeability anisotropy ratio K^*makes it possible to quickly reach the threshold of convection;
- We obtained the same effect for the permeability anisotropy ratio ζ;
- On the other hand, the threshold of the convection increases when we increase the orientation angle φ of the principal axes of permeability. In other words, when the principal directions of permeability coincide with the directions of the principal axes $(\varphi = 90°)$, the threshold of the convection is quickly reached by cons when the principal directions of permeability are confused with the principal axis (o, x) that is $(\varphi = 0°$, the threshold of the convection is high.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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