



# Velocity Slip Effect on MHD Power-Law Fluid over a Moving Surface with Heat Generation, Viscous Dissipation and Thermal Radiation

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## Authors' contributions

This work was carried out in collaboration among between authors. Both authors read and approved the final manuscript.

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## ABSTRACT

The problem of laminar boundary layer flow of power-law fluid over a continuous moving surface in the presence of a transverse magnetic field with velocity slip was investigated. The governing partial differential equations for the flow and heat transfer were transformed into non-linear ordinary differential equations using the similarity method. These equations were solved numerically by applying the fourth-order Runge-Kutta method with a shooting technique. The solution is found to be dependent on various parameters such as power-law index, magnetic field parameter, suction, and injection parameters. The effect of various flow parameters in the form of dimensionless quantities on the flow field is discussed and graphically presented. It was observed that an increase in the magnetic property results to a decrease flow of fluid velocity and also, an increase in the Prandtl number results to an increase in the rate of heat transfer.

**Keywords:** Power-law; viscous dissipation; velocity slip, MHD and thermal radiation.

## 1. INTRODUCTION

Over the years, analysis of the boundary layer concept in flow of non-Newtonian or power-law

fluids has been of utmost interest in research owing to their industrial and numerous engineering applications [1-8]. Among others, we

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can name food processing, petroleum production, metallurgy, and aerodynamic extrusion of plastic sheets, manufacture of polymeric sheets, paper production and textile industries. Many of the fluid encountered in real life followed the empirical Ostward-de-Waele power-law model. In describing the behaviour of fluids, this model permits mathematical predictions and correlate experimental data. The boundary layer flow of viscous incompressible fluid on moving surface with constant velocity was first studied by Sakiadis (1967).

Dharmendar Reddy et al. [9] investigated the effect of thermal radiation and viscous dissipation on MHD boundary layer flow and heat transfer over a porous exponentially stretching sheet. A numerical technique known as Keller-box method was applied to determine the solutions of the governing non-linear ordinary differential equations. It was revealed from their investigation that temperature profile decreases as suction parameter increases. If the permeability parameter rises, the surface shear stress rises as well. With increasing Prandtl number, temperature decreases. The effect of the suction as well as magnetic parameter on fluid is to suppress the velocity field which in turn causes the enhancement of the skin friction coefficient.

The effects of surface slip and heat generation on the flow and heat transfer of a non-Newtonian power-law fluid on a continuously moving surface was studied by Mahmoud M.A. [10]. In his investigation, it was verified that the local skin friction coefficient decreased as the slip parameter increased. Also, it was found that the Nusselt number decreases as the slip parameter or heat generation parameter increased and the heat absorption parameter has the effect of increasing Nusselt number.

The results of thermal radiation and heat transfer over an unsteady stretching surface embedded in a porous medium in the presence of a heat source or sink were investigated by Elbashbeshy and Aldawody [11], who found that the velocity decreased as the unsteadiness parameter increased, and the temperature decreased as the radiation parameter and Prandtl number increased although the rate of heat transfer increases as the heat source increases.

Cortell R. [12] investigated internal heat generation and radiation effects on a certain free convection flow. The paper investigates

momentum and heat transfer in a continuous free convection flow over a vertical, permeable, semi-infinite flat plate of variable temperature immersed in a fluid saturated porous medium at ambient temperature.

The result shows that an increase in suction parameter decreases the temperature whereas; an increase in injection parameter increases the temperature distribution in the flow region.

Zhang Z and Wang J [13] investigated the steady laminar flow of a non-Newtonian fluid obeying the power-law model over a stretching surface. They concluded that the problem has a unique normal solution for  $0 < n < 1$  and has a unique generalised normal solution for  $n > 1$ .

B. Shashidar Reddy et al [14] investigated MHD boundary layer flow of a non-Newtonian power-law fluid on a moving flat plate. As the plate and the fluid travel in opposite directions near the separation field, they discovered that a dual solution occurs. Also, it was evident that the effect of magnetic field parameter reduces the velocity profile far away from the plate and reverse phenomenon is observed near the plate. In the recent work B.K Swain *et al* [15] velocity profile diminishes when the magnetic parameter increases and also the rate of heat transfer diminishes as thickness leads to low heat transfer coefficient. However, the purpose of the present investigation is to look at the effects of velocity slip on MHD power-law fluid over a moving surface with heat generation.

## 2. FORMULATION OF THE PROBLEM AND GOVERNING EQUATION

Consider a laminar, steady, two dimensional, radiating, viscous, incompressible boundary layer flows and heat transfer over a moving permeable surface with constant velocity through non-Newtonian power-law fluid at rest.

The steady two dimensional boundary layer equations taking into account the viscous dissipation, thermal radiation effect, heat generation (absorption) and magnetic effect in the energy equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\rho} \frac{\partial}{\partial y} \left( \left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y} \right) - \frac{v_u}{k} - \frac{\sigma B^2 u}{\rho} \tag{2.2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p} \left| \frac{\partial u}{\partial y} \right|^{n+1} + \frac{Q_0(T-T_\infty)}{\rho c_p} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} \quad (2.3)$$

Subject to the following boundary condition

$$y = 0: \quad u = U + S_1 \left( \left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y} \right), \quad v = -v_w, \quad (2.4)$$

$$y \rightarrow \infty \quad u \rightarrow 0, \quad T = T_\infty \quad (2.5)$$

Where  $u$  and  $v$  are the velocity components along the  $x$  and  $y$  axes,  $\rho, \alpha, \nu$  and  $\mu$  are the fluid density, thermal diffusivity, kinematic viscosity of the fluid and the consistency index for non-Newtonian viscosity,  $c_p$  is the specific heat at constant pressure and  $T$  is the temperature of the fluid,  $n$  is the power-law index,  $T_w$  is the temperature of the plate,  $T_\infty$  is the temperature of the fluid far away from the plate,  $v_w$  is the constant velocity normal to the wall and  $S_1$  is the slip coefficient having dimension of length,  $q_r$  is the radiative heat flux.

Using Rosseland approximation to radiation we

$$\text{write } q_r = \frac{-4\sigma^* \partial T^4}{3k^* \partial y} \quad (2.6)$$

Where  $\sigma^*$  is the Stephan-Boltzmann constant,  $k^*$  is the absorption coefficient in the above equation. We assume that the temperature difference within the flow is such that the term  $T^4$  may be expressed as a linear function of temperature. Expanding  $T^4$  in a Taylor series about  $T_\infty$  and then neglecting higher order terms, we can write

$$T^4 \cong 4T_\infty^3 - 3T_\infty^4 \quad (2.7)$$

Therefore, equation (2.6) becomes

$$q_r = \frac{-16\sigma^* T_\infty^3 \partial T}{3k^* \partial y} \quad (2.8)$$

Proceeding with the analysis, the following dimensionless variables are introduced

$$\eta = y \left( \frac{U^2 - n}{\nu x} \right)^{\frac{1}{n+1}} \quad (2.9a)$$

$$\psi = (\nu x U^{2n-1})^{\frac{1}{n+1}} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (2.9b)$$

Where  $\nu = \frac{\mu}{\rho}$  is the kinematic viscosity,  $f(\eta) = \frac{\psi(\eta)}{(\nu x U^{2n-1})^{\frac{1}{n+1}}}$  is the dimensionless stream function,  $\theta$  is the dimensionless temperature and  $\psi$  is the stream function that satisfies Eq. (2.1) and is defined as

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (2.10)$$

Using these transformations, the governing equations (1) – (3) are transformed to the following set of local similarity equations:

$$n(n+1)|f''|^{n-1}f''' + ff'' - Mf' = 0 \quad (2.11)$$

$$\left(1 + \frac{4}{3R}\right)\theta'' + PrEc|f''|^{n+1} + QPr\theta + \frac{Pr}{n+1}f\theta' = 0 \quad (2.12)$$

Where  $M = \frac{\sigma B_0^2}{\rho \nu} x$ ,  $Pr = \frac{U}{\alpha x} \left( \frac{U^2 - n}{\nu x} \right)^{\frac{-2}{n+1}}$  is the modified non-Newtonian Prandtl number and  $Ec = \frac{U^2}{c\rho(T_w - T_\infty)}$  is the Eckert number,  $R = \frac{k^*k}{4\sigma^*T_\infty^3}$  is the radiation parameter,  $Q = \frac{xQ_0}{U\rho c_p}$  is the heat generation ( $Q > 0$ ) or heat absorption ( $Q < 0$ ) and  $f_w = \frac{(n+1)x^{\frac{n}{n+1}}v_w}{\nu^{\frac{1}{n+1}}U^{\frac{2n-1}{n+1}}}$  is the suction parameter ( $f_w > 0$ ) or the injection parameter ( $f_w < 0$ ).

The transformed boundary condition (2.4) and (2.5) are:

$$\eta = 0 \quad f = f_w \quad f' = 1 + S(f''|^{n-1}f''), \quad \theta = 1 \quad (2.13a)$$

$$\eta \rightarrow \infty \quad f' \rightarrow 0 \quad \theta \rightarrow 0 \quad (2.13b)$$

$$\text{But } S = \frac{S_1}{U} \left( \frac{U^3}{\nu x} \right)^{\frac{n}{n+1}} \text{ is the slip parameter} \quad (2.14)$$

The physical quantities of interest are the skin coefficient  $C_{fx}$  and the Nusselt number  $Nu_x$ . The local skin friction is expressed as

$$C_{fx} = -\frac{2\tau_w}{\rho U^2} = -2Re_x^{\frac{-1}{n+1}}|f''(0)|^{n-1}f''(0) \quad (2.15)$$

$$\text{Where Reynold number. } Re_x = \frac{\rho U^{2-n} x^n}{\mu} \quad (2.16)$$

$$\text{The Nusselt number } Nu_x = \frac{x \left( \frac{\partial T}{\partial y} \right)_{y=0}}{(T_w - T_\infty)} = -Re_x^{\frac{1}{n+1}} \cdot \theta'(0) \quad (2.17)$$

### 3. NUMERICAL SOLUTION

The set of non-linear differential equations (12) and (13) with the boundary conditions (13a) and (13b) re transformed into initial value problems. The equations form a three point BVP and do not admit a closed form solution and therefore it is solved numerically using fourth-order Runge-Kunta method. In this method, the governing equations as well as the boundary conditions are first converted into first order system as follows.

$$\begin{aligned}
 y_1 &= f & y_2 &= y'_1 & y_3 &= y'_2 & y'_3 &= \\
 -\frac{1}{n(n+1)}y_1y_3|y_3|^{1-n} + My_2 & & y'_4 &= y_5 & y'_5 &= \\
 \left(\frac{3R}{3R+4}\right)(-PrEc|y_3|^{n+1} - \frac{Pr y_1 y_5}{n+1} - QPr y_4) & & & & & & & 
 \end{aligned}$$

### 4. RESULT AND DISCUSSION

Using the numerical method mentioned in the previous section, numerical computations are carried out at different values of magnetic parameter, radiation parameter, and power-law index, Prandtl number, and Eckert number. The non-dimensional parameters described in the problem were given numerical values to have a true picture of the physical problem. Fig. 1 and Table 1 show the effect of slip on the velocity profile. It is seen that fluid velocity decreases as the slip parameter increases. An increase in the slip parameter decreases the boundary layer thickness but it causes an increase in temperature as found in Fig. 2 and Table 2.

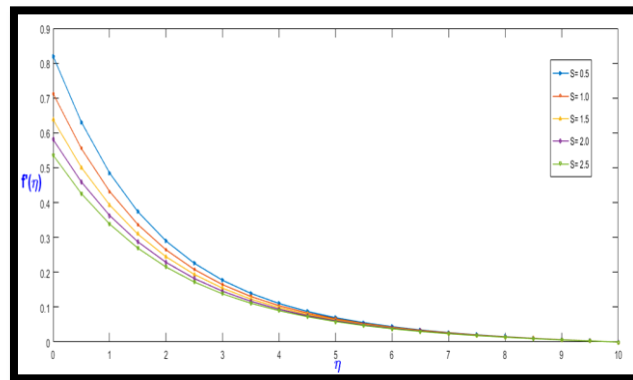


Fig. 1. Velocity profile  $F'(\eta)$  vs.  $\eta$  for different values of slip parameter  $S$

Table 1. Table of value showing the effect of velocity profile with varying slip parameter

$\eta$	$F'(\eta)$ when $S = 0.5$	$F'(\eta)$ when $S = 1.0$	$F'(\eta)$ when $S = 1.5$	$F'(\eta)$ when $S = 2.0$	$F'(\eta)$ when $S = 2.5$
0	0.8203	0.7127	0.6377	0.581	0.5361
0.5	0.631	0.5551	0.5009	0.4593	0.4259
1	0.4849	0.4321	0.3933	0.363	0.3384
1.5	0.3738	0.3371	0.3095	0.2875	0.2693
2	0.2896	0.2642	0.2444	0.2284	0.215
2.5	0.2258	0.208	0.1938	0.1821	0.1721
3	0.177	0.1645	0.1543	0.1457	0.1382
3.5	0.1396	0.1307	0.1233	0.1169	0.1113
4	0.1105	0.1042	0.0988	0.094	0.0898
4.5	0.0878	0.0833	0.0793	0.0757	0.0726
5	0.0698	0.0666	0.0637	0.061	0.0586
5.5	0.0556	0.0532	0.0511	0.0491	0.0472
6	0.0441	0.0424	0.0408	0.0393	0.0379
6.5	0.0348	0.0336	0.0324	0.0313	0.0302
7	0.0271	0.0263	0.0254	0.0246	0.0238
7.5	0.0208	0.0202	0.0196	0.019	0.0184
8	0.0154	0.015	0.0146	0.0142	0.0138
8.5	0.0108	0.0106	0.0103	0.01	0.0097
9	0.0068	0.0067	0.0065	0.0064	0.0062
9.5	0.0033	0.0032	0.0031	0.003	0.003
10	0	0	0	0	0

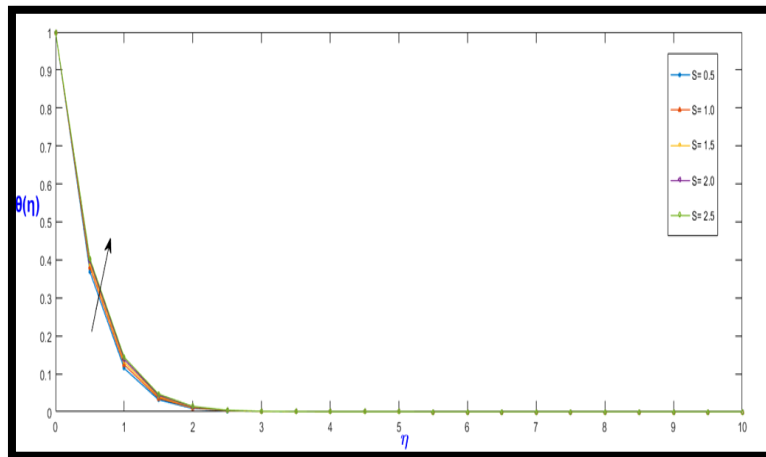


Fig. 2. Velocity profile  $\theta(\eta)$  vs  $\eta$  for different values of slip parameter S

Table 2. Table of value showing the effect of temperature profile with varying slip parameter

$\eta$	$\theta(\eta)$ when $S = 0.5$	$\theta(\eta)$ when $S = 1.0$	$\theta(\eta)$ when $S = 1.5$	$\theta(\eta)$ when $S = 2.0$	$\theta(\eta)$ when $S = 2.5$
0	1	1	1	1	1
0.5	0.3684	0.38	0.3892	0.3968	0.4032
1	0.1144	0.1237	0.1313	0.1379	0.1436
1.5	0.0318	0.0361	0.04	0.0434	0.0465
2	0.0086	0.0101	0.0115	0.0128	0.0141
2.5	0.0025	0.0029	0.0033	0.0038	0.0042
3	0.0009	0.001	0.0011	0.0012	0.0013
3.5	0.0004	0.0004	0.0004	0.0005	0.0005
4	0.0002	0.0002	0.0002	0.0002	0.0002
4.5	0.0001	0.0001	0.0001	0.0001	0.0001
5	0.0001	0.0001	0.0001	0.0001	0.0001
5.5	0	0	0	0	0
6	0	0	0	0	0
6.5	0	0	0	0	0
7	0	0	0	0	0
7.5	0	0	0	0	0
8	0	0	0	0	0
8.5	0	0	0	0	0
9	0	0	0	0	0
9.5	0	0	0	0	0
10	0	0	0	0	0

An inverse relationship was observed comparing the magnetic field parameter and the velocity profile. In Fig 3 it was evident that an increase in the magnetic field parameter decreases the velocity profile as well as the skin friction in Fig 4. The velocity profile increases as the magnetic property decreases. However, the same is not the case for the temperature distribution. Increase in the magnetic field parameter shows increase the temperature distribution of the fluid. This is evident as shown in Fig 5. At a larger distance away from the wall, the temperature profile remains the same

even as the transverse magnetic parameter increases.

A direct relationship exists between temperature profiles and the rate of heat transfer as the radiation parameter increases as evident in Fig 6 and 7. Also for increase in Prandtl number, there is an observable increase in the rate of heat transfer and decrease in temperature profile as shown in Figs 8 and 9. Equally, an increase in Eckert parameter result to an increase in the temperature profile and a considerable decrease in the rate of heat transfer as shown in Figs 10 and 11.

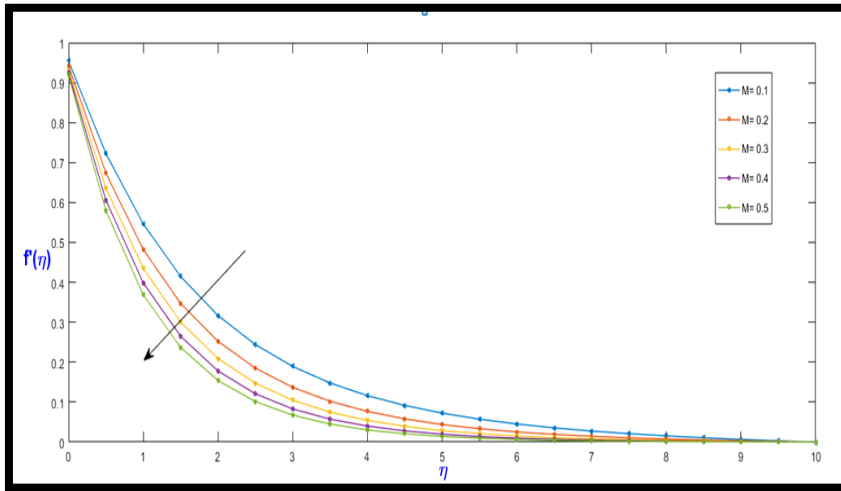


Fig. 3. Velocity profile vs. magnetic parameter

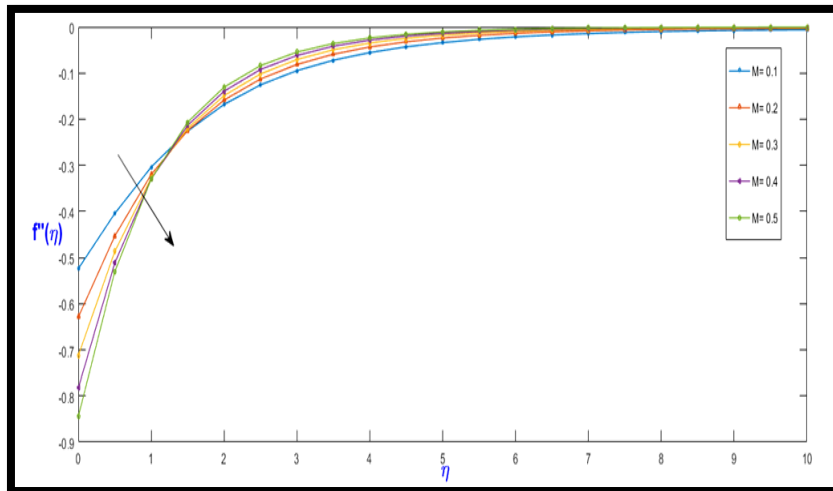


Fig. 4. Skin friction vs. magnetic parameter

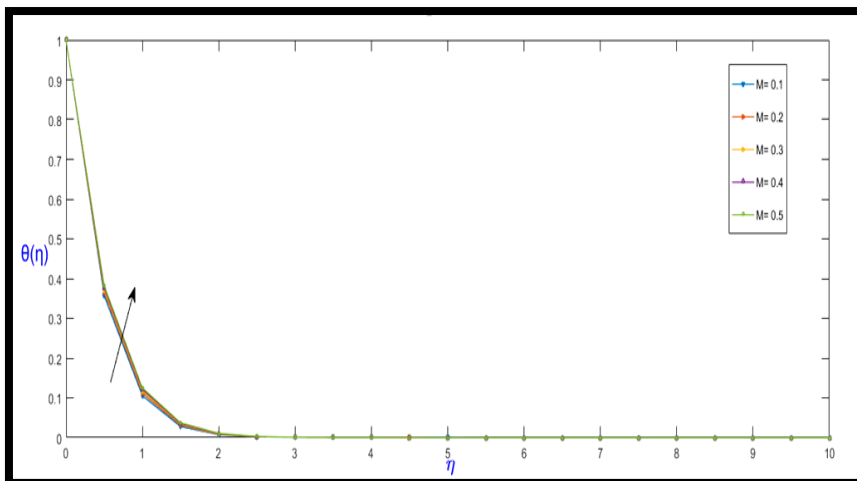


Fig. 5. Temperature distribution vs. magnetic parameter

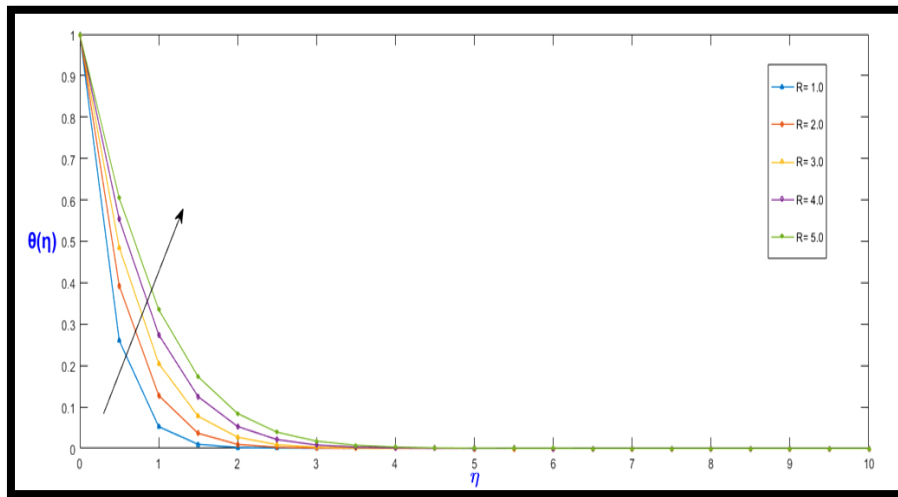


Fig 6. Temperature profile vs. radiation parameter

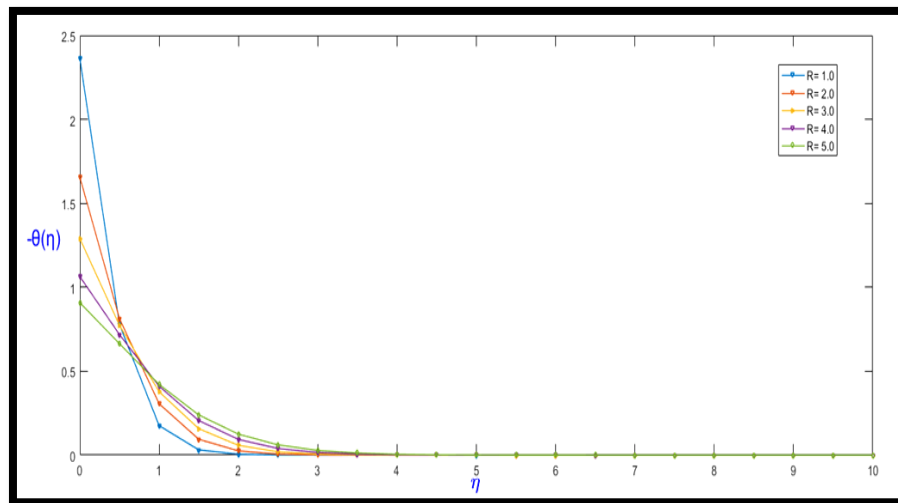


Fig. 7. Rate of heat transfer profile vs. radiation parameter

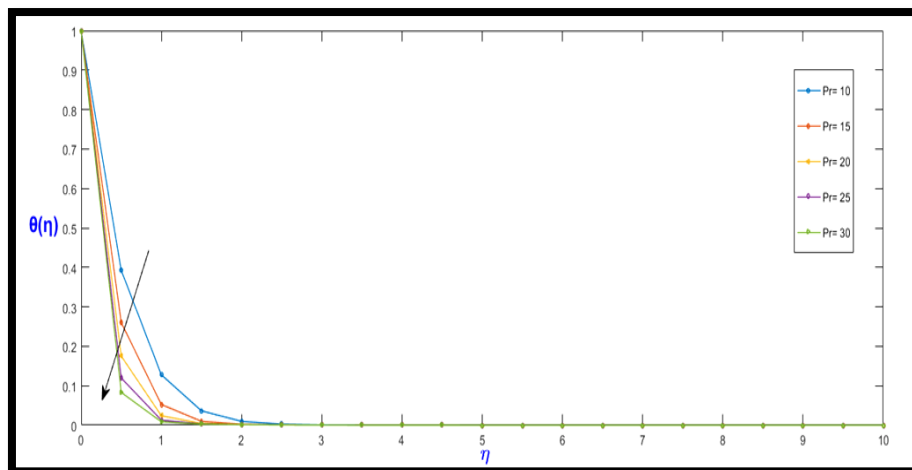


Fig. 8. Temperature profile vs. prandtl number

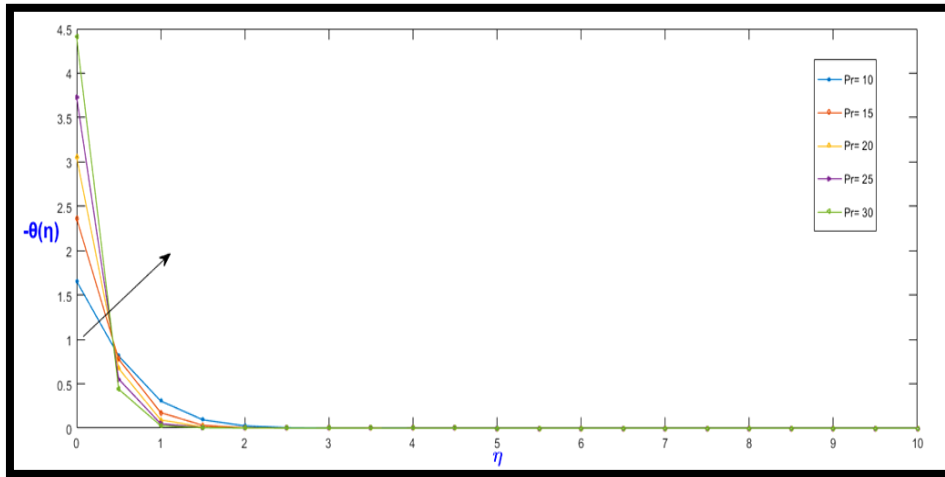


Fig. 9. Rate of heat transfer profile vs. Prandtl number

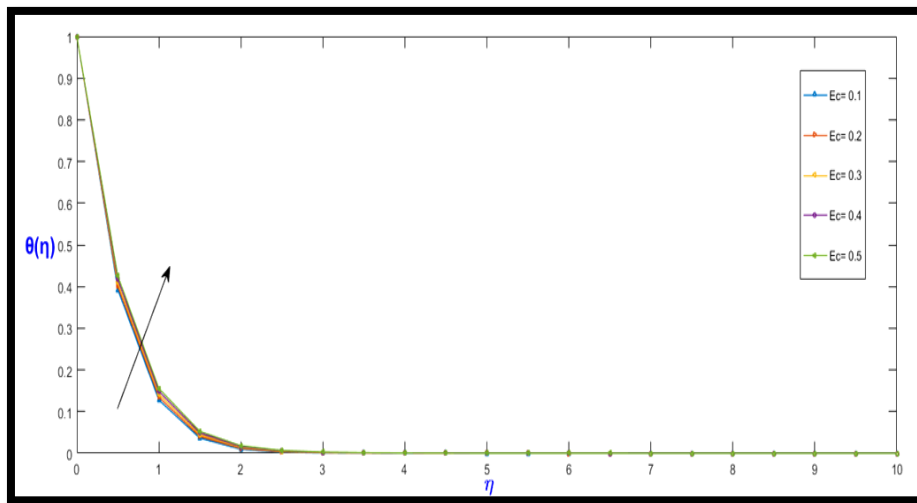


Fig. 10. Temperature profile vs. eckert number

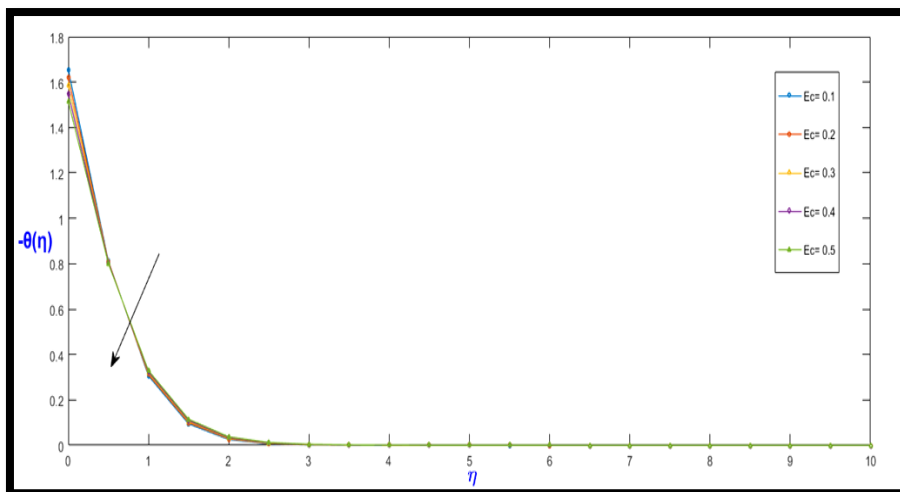


Fig. 11. Rate of heat transfer profile vs. eckert number



**Table 3. Values of the heat transfer coefficient,  $-\theta'(0)$  for different values of  $Pr$  when  $M = R = Ec = 0$** 

$Pr$	Magyari and Keller (2000)	El-Aziz [1]	Bidin and Nazar [17]	Mukhopadhyay and Gorla (2009)	Ishak [16]	Sreenivasulu and Bhaskar [2]	Present Results
1	0.954782	0.954785	0.9547	0.9547	0.9548	0.95495	0.9547
2			1.4714	1.4714	1.4715	1.47142	1.4715
3	1.869075	1.869074	1.8691	1.8691	1.8691	1.86904	1.8690
5	2.500135	2.500132		2.5001	2.5001	2.50011	2.5001
10	3.660379	3.660372		3.6604	3.6604	3.66035	3.6604

The present result was compared with the results of Magyari and Keller (2000), Ishak [16], Bidin and Nazar [17], El-Aziz [1], Mukhopadhyay and Gorla (2009), and Sreenivasulu and Bhaskar [2]. The results from the present work for the heat transfer coefficient are in good agreement with the works cited. See Table 1.

## 5. CONCLUSION

In the present study we have investigated the velocity slip effect on MHD non-Newtonian fluid flowing over a surface with consideration for viscous dissipation, heat generation and adsorption and thermal radiation. From this investigation, velocity slip and transverse magnetic parameter have the effect to decrease the velocity profile and reducing the boundary layer thickness. In similitude, an increase in the power-law index,  $n$ , leads to thinning of the thermal boundary layer thickness. The magnetic parameter causes an increase in temperature profile and the rate of heat transfer thereby increasing the thermal boundary layer thickness.

Also, there is an observable increase temperature profile and the rate of heat transfer as the radiation parameter increases. For increment in Prandtl number, this results to increase in the rate of heat transfer. In the same light, it was observed that an increase in the Prandtl number results to decreased temperature profile. I was so clear that a reverse result was observed when the Eckert number increases. Increase in Eckert number results to increase in temperature profile and decrease in the heat transfer rate.

## COMPETING INTERESTS

Authors have declared that no competing interests exist.

## REFERENCES

1. El-Aziz MA. Viscous dissipation effect on mixed convection flow of a micropolar fluid over an exponentially stretching sheet. *Canadian Journal of Physics*. 2009;87:359-368.
2. Sreenivasulu P, Poornima T, Bhaskar Reddy N. Thermal radiation on MHD boundary layer slip flow past a permeable exponential stretching sheet in the presence of Joule heating and viscous dissipation. *Journal of Applied Fluid Mechanics*. 2016;9(1):267-278.
3. Sakiadis BC. "Boundary layer behaviour on continuous solid surfaces: Boundary layer equations for two dimensional and axis symmetric flow." *ALCHE Journal*. 1961;7:26-28.
4. Mahmoud MAA, Mahmoud MAE. "Analytical solutions of hydro-magnetic boundary layer flow of a non-Newtonian power-law fluid past a continuously moving surface." *Acta Mech*. 2006;181:83-89.
5. Sahu AK, Mathur MN, Chaturani P, Saena Bharatiya S. "Momentum and heat transfer from a continuous moving surface to a power-law fluid." *Acta Mech*. 2000;142:119-131.
6. Wang C, Pop I. "Analysis of the flow of a power-law fluid on an unsteady stretching surface by means of homotopy analysis method." *Journal Non-Newtonian fluid mechanics*. 2006;138.
7. Tamana Sultana, Sumon Saha. In a porous medium over a stretching surface with internal heat generation and suction or injection in the presence of radiation." *Journal of Mechanical Engineering*. 2009;40(1).
8. Mukhopadhyay S, Uddin MDS, Layek GC. Lie group analysis of MHD boundary layer slip flow past a heated stretching sheet in presence of heat source/sink. *International*

- Journal of Applied Mathematics and Mechanics. 2012;8(16):51-66.
9. Dharmandar Reddy Srinivasarao V, Ramya D. Effect of thermal radiation and viscous dissipation on MHD boundary layer flow and heat transfer over a porous exponentially stretching sheet. Open Journal of Applied and Theoretical Mathematics. 2016;2(4):731-752.
  10. Mahmoud MA. Slip velocity effect on non-Newtonian power-law fluid over a moving permeable surface with heat generation. Journal of Mathematical and Computer Modeling. 2011;54:1228-1237
  11. Elbashbeshy EMA, Aldawody DA. Effect of thermal radiation and magnetic field on unsteady mixed convection flow and heat transfer over a porous stretching surface in the presence of internal heat generation/absorption. International Journal of the physical science. 2011;6(6):1540-1548.
  12. Rafael Cortell. Internal heat generation and radiation effect on a certain free convection flow. International Journal of Nonlinear Science. 2010;9(4):468-479.
  13. Zhang Z, Wang J. "The similarity solution of MHD flow of a power-law fluid over a stretching sheet". J. Math. Anal. Appl. 2007;330.
  14. Shashidar Reddy B, Kishan N, Rajasekhar MN. MHD boundary layer of a non-Newtonian power-law fluid on a moving falt plate. Advances in Applied Science Research. 2012;3(3):1472-1481.
  15. Swain BK, Parida BC, Karc S, Senapati N. Viscous dissipation and joule heating effect on MHD flow and heat transfer past a stretching sheet embedded in a porous medium. Heliyon. 2020;6(10).
  16. Ishak A. MHD boundary layer flow due to an exponentially stretching sheet with radiation effect. Sains Malaysiana. 2011;40:391-395.
  17. Bidin B, Nazar R. Numerical solution of the boundary layer flow over an exponentially stretching sheet with thermal radiation. European Journal of Scientific Research. 2009;33(4):710-717.

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