
**British Journal of Mathematics & Computer Science**
**13(1): 1-10, 2016, Article no.BJMCS.21820**
***ISSN: 2231-0851***

**SCIENCEDOMAIN** *international*
www.sciencedomain.org

SCIENCEDOMAIN

# New Fourth-order Schroder-type Methods for Finding Zeros of Nonlinear Equations Having Unknown Multiplicity

**R. Thukral[1*]**

[1]*Padé Research Centre, 39 Deanswood Hill, Leeds, West Yorkshire, LS17 5JS, England.*

***Article Information***

DOI: 10.9734/BJMCS/2016/21820
*Editor(s):*
(1) Carlo Bianca, Laboratoire de Physique Théorique de la Matière Condensée, Sorbonne Universités, France.
*Reviewers:*
(1) Grienggrai Rajchakit, Maejo University, Thailand.
(2) Young Hee Geum, Dankook University, South Korea.
(3) Manoel Borges, São Paulo State University, Brazil.
(4) Gustavo Fernández-Torres, National Autonomous University of México, Mexico.
Complete Peer review History: http://sciencedomain.org/review-history/12331

***Original Research Article***

*Received: 04 September 2015*
*Accepted: 08 October 2015*
*Published: 18 November 2015*

## Abstract

In this paper we define two new fourth-order Schroder-type methods for finding zeros of nonlinear equations having unknown multiplicity. In terms of computational cost the new iterative methods requires six evaluations of functions per iteration. It is proved that the new methods have a convergence of order four. Numerical comparisons are included to demonstrate exceptional convergence speed of the proposed methods.

*Keywords: Schroder-type method; root-finding; nonlinear equations; multiple roots; order of convergence; efficiency index.*

**Subject classifications:** AMS (MOS): 65H05, 41A25.

## 1 Introduction

The solution of a nonlinear equation is one of the most important problem in computational mathematics, science and engineering [1,3,10]. In this work, we present two new fourth-order iterative methods to find multiple roots of the nonlinear equation $f(x)=0$, where $f : I \subset \mathbb{R} \to \mathbb{R}$ for an open interval $I$ is a scalar function. The multi-point method is of great practical importance since it overcomes theoretical limits of one-point methods concerning convergence order and computational efficiency. Recently, many modifications of the Newton-type methods for simple roots have been proposed and examined [3] but little

**Corresponding author: E-mail: rthukral@hotmail.co.uk;*

work has been done on multiple roots. Therefore, in this study, we are interested in the case that $\alpha$ is a root of multiplicity $m > 1$ of a nonlinear equation, that is $f^{k}(\alpha) = 0,\ k = 0,1,2...m-1$ and $f^{m}(\alpha) \neq 0.$ Therefore, the objective of this study is to develop a new class of iterative method for finding multiple roots of nonlinear equations of a higher order than the existing iterative methods [3-9]. The purpose of this paper is to show further development of the third-order methods [9]. In addition, the new iterative methods have a better precision than the classical Schroder method and Soleymani et al. methods [5-7]. Hence, the proposed fourth-order method is significantly better when compared with these established methods.

The summary of the paper is as follows. The essential definitions relevant to the present study are given in section 2. In section 3, we derive two new multi-point methods and verify their convergence order. In section 4, two well-known methods are stated, which will illustrate the effectiveness of the new fourth-order iterative methods. Finally, in section 5, numerical examples are given to show the performance of the new Schroder-type methods.

# 2 Basic Definitions

Here we shall state some of the essential definitions. These definitions will determine the behaviour of the method, [1,3,9,10].

**Definition 1.** Let $f(x)$ be a real-valued function with a root $\alpha$ and let $\{x_n\}$ be a sequence of real numbers that converge towards $\alpha.$ The order of convergence *p* is given by

$$\lim_{n\to\infty} \frac{x_{n+1} - \alpha}{(x_n - \alpha)^p} = \zeta \neq 0, \tag{1}$$

where $p \in \mathbb{R}^{+}$ and $\zeta$ is the asymptotic error constant [1,3,10].

**Definition 2.** Let $e_k = x_k - \alpha$ be the error in the *k*th iteration, then the relation

$$e_{k+1} = \zeta e_k^p + \mathrm{O}\left(e_k^{p+1}\right), \tag{2}$$

is the error equation. If the error equation exists, then *p* is the order of convergence of the iterative method [1,3,10].

**Definition 3.** Let *r* be the number of function evaluations of the method. The efficiency of the method is measured by the concept of efficiency index and defined as:

$$\sqrt[r]{p}, \tag{3}$$

where *p* is the order of convergence of the method [1,3,10].

**Definition 4.** Suppose that $x_{n-2}, x_{n-1}$ and $x_n$ are three successive iterations closer to the root $\alpha$ . Then the computational order of convergence may be approximated by

$$\mathrm{COC} \approx \frac{\ln\left|\delta_n \div \delta_{n-1}\right|}{\ln\left|\delta_{n-1} \div \delta_{n-2}\right|} \tag{4}$$

where $\delta_i = f\left(x_i\right) \div f'\left(x_i\right)$, [9].

# 3 Construction of the New Methods and Convergence Analysis

In this section we derive two new fourth-order multi-point methods for finding multiple roots of a nonlinear equation. In terms of computational cost the new iterative methods require total of six function evaluations. The new Schroder-type methods are actually based on the Schroder second-order method and Thukral third-order method given in [5,9], respectively.

## 3.1 Method 1

The first of our new scheme is actually based on the classical Schroder method. We use this method as our first step and repeat the process at an improved point, therefore the new scheme is expressed as:

$$y_n = x_n - \frac{f\left(x_n\right) f'\left(x_n\right)}{f'\left(x_n\right)^2 - f\left(x_n\right) f''\left(x_n\right)}, \tag{5}$$

$$x_{n+1} = y_n - \frac{f\left(y_n\right) f'\left(y_n\right)}{f'\left(y_n\right)^2 - f\left(y_n\right) f''\left(y_n\right)}, \tag{6}$$

where $n \in \mathbb{N}$, $x_0$ is the initial guess and provided that the denominator of (6) is nonzero.

**Theorem 1**

Let $\alpha \in I$ be a multiple root of multiplicity *m* of a sufficiently smooth function $f : I \subset \mathbb{R} \to \mathbb{R}$ for an open interval *I*. If the initial guess $x_0$ is sufficiently close to $\alpha$, then the convergence order of method defined by (6) is four.

**Proof**

Let $\alpha \in \mathbb{R}$ be a multiple root of multiplicity *m* of a sufficiently smooth function $f\left(x\right)$, $e = x - \alpha$, $\hat{e} = y - \alpha,$

Expanding $f\left(x\right), f'\left(x\right), f''\left(x\right), f'''\left(x\right), f\left(y\right), f'\left(y\right), f''\left(y\right),$ in a Taylor's series about $\alpha$ , we have

$$f(x_n) = \left(\frac{f^{(m)}(\alpha)}{m!}\right) e_n^m \left[1 + \sum_{i=1}^{\infty} A_i e_n^i\right], \tag{7}$$

$$f'(x_n) = \left(\frac{f^{(m)}(\alpha)}{(m-1)!}\right) e_n^{m-1} \left[1 + \sum_{i=1}^{\infty} B_i e_n^i\right], \tag{8}$$

$$f''(x_n) = \left(\frac{f^{(m)}(\alpha)}{(m-2)!}\right) e_n^{m-2} \left[1 + \sum_{i=1}^{\infty} C_i e_n^i\right], \tag{9}$$

$$f'''(x_n) = \left(\frac{f^{(m)}(\alpha)}{(m-3)!}\right) e_n^{m-3} \left[1 + \sum_{i=1}^{\infty} D_i e_n^i\right], \tag{10}$$

$$f(y_n) = \left(\frac{f^{(m)}(\alpha)}{m!}\right) \hat{e}_n^{m} \left[1 + \sum_{i=1}^{\infty} A_i \hat{e}_n^i\right], \tag{11}$$

$$f'(y_n) = \left(\frac{f^{(m)}(\alpha)}{(m-1)!}\right) \hat{e}_n^{m-1} \left[1 + \sum_{i=1}^{\infty} B_i \hat{e}_n^i\right], \tag{12}$$

$$f''(y_n) = \left(\frac{f^{(m)}(\alpha)}{(m-2)!}\right) \hat{e}_n^{m-2} \left[1 + \sum_{i=1}^{\infty} C_i \hat{e}_n^i\right], \tag{13}$$

where $n \in \mathbb{N}$ and

$$T_k = \frac{f^{(m+k)}(\alpha)}{f^{(m)}(\alpha)}, \tag{14}$$

$$A_k = \frac{m!T_k}{(m+k)!}, \tag{15}$$

$$B_k = \frac{(m-1)!T_k}{(m+k-1)!}, \tag{16}$$

$$C_k = \frac{(m-2)!T_k}{(m+k-2)!}, \tag{17}$$

$$D_k = \frac{(m-3)!T_k}{(m+k-3)!}. \tag{18}$$

Dividing (7) by (8), we get

$$\frac{f(x_n)}{f'(x_n)} = \frac{e_n}{m} - \frac{T_1 e_n^2}{m^2(m+1)} + \frac{\left(T_1^2(m+2) - 2mT_2\right)}{m^3(m+1)(m+2)} e_n^3 + \cdots \tag{19}$$

and dividing (9) by (8) yields

$$\frac{f''(x_n)}{f'(x_n)} = \frac{m-1}{e_n} - \frac{T_1}{m} + \frac{\left(T_1^2(m+1) - 2mT_2\right)}{m^2(m+1)} e_n + \cdots \tag{20}$$

Product of (19) and (20), we obtain

$$\left(\frac{f(x_n)}{f'(x_n)}\right)\left(\frac{f''(x_n)}{f'(x_n)}\right) = \frac{m-1}{m} - \frac{2T_1 e_n}{m^2(m+1)} + \frac{\left(3T_1^2(m+2) - 2mT_2\right)}{m^3(m+1)(m+2)} e_n^2 + \cdots \tag{21}$$

Substituting appropriate expressions in (5) and simplifying, yields

$$y_n - \alpha = x_n - \frac{f(x_n) f'(x_n)}{f'(x_n)^2 - f(x_n) f''(x_n)} = -\frac{T_1 e_n^2}{m(m+1)} + \cdots \tag{22}$$

Now we need to expand $f(y_n), f'(y_n), f''(y_n)$ about $\alpha$. Replacing appropriate expressions in (6),

$$e_{n+1} = y_n - \alpha - \frac{f(y_n) f'(y_n)}{f'(y_n)^2 - f(y_n) f''(y_n)}. \tag{23}$$

Consequently, we obtain

$$e_{n+1} = \left(\frac{\left(T_1^3\right)}{m^3(m+1)^3}\right) e_n^4, \tag{24}$$

which indicates that the order of convergence of the new Schroder-type method defined by (6) is four. This completes the proof.

## 3.2 Method 2

In this sub-section we derive another new fourth-order multi-point iterative method for finding multiple roots of a nonlinear equation. We construct the new method by using Thukral third-order method as our first step and the second step is combination of Newton-Schroder principle. Hence, the second new method is given as:

$$y_n = x_n - \frac{2\left(f(x_n) f'(x_n)^2 - f(x_n)^2 f''(x_n)\right)}{2f'(x_n)^3 - 3f(x_n) f'(x_n) f''(x_n) + f'''(x_n) f(x_n)^2} \tag{25}$$

$$x_{n+1} = y_n - \left( \frac{f'(x_n)^2}{f'(x_n)^2 - f(x_n) f''(x_n)} \right) \left( \frac{f(y_n)}{f'(y_n)} \right), \tag{26}$$

where $n \in \mathbb{N}$, as before $x_0$ is the initial guess and provided that the denominator of (26) is nonzero.

**Theorem 2**

Let $\alpha \in I$ be a multiple root of multiplicity *m* of a sufficiently smooth function $f : I \subset \mathbb{R} \to \mathbb{R}$ for an open interval *I*. If the initial guess $x_0$ is sufficiently close to $\alpha$, then the convergence order of method defined by (26) is four.

**Proof**

Using the Taylor series expansion expressions given in the proof of the theorem 1 and substituting them into (25).

$$y_n - \alpha = x_n - \frac{2\left( f(x_n) f'(x_n)^2 - f(x_n)^2 f''(x_n) \right)}{2 f'(x_n)^3 - 3 f(x_n) f'(x_n) f''(x_n) + f'''(x_n) f(x_n)^2}. \tag{27}$$

We obtain

$$y_n - \alpha = -\frac{\left( T_1^2 (m+2) - 2(m+1) T_2 \right)}{m(m+1)^2 (m+2)} e_n^3 + \cdots \tag{28}$$

Expanding $f(y_n), f'(y_n)$ about $\alpha$, and substituting them in (26), we attain

$$e_{n+1} = y_n - \alpha - \left( \frac{f'(x_n)^2}{f'(x_n)^2 - f(x_n) f''(x_n)} \right) \left( \frac{f(y_n)}{f'(y_n)} \right). \tag{29}$$

Simplifying (29), we obtain

$$e_{n+1} = 2T_1 \left( \frac{\left( 2(m+1) T_2 - (m+2) T_1^2 \right)}{m^2 (m+1)^3 (m+2)} \right) e_n^4. \tag{30}$$

which indicates that the order of convergence of the new Schroder-type method defined by (26) is four. This completes the proof.

# 4 The Soleymani et al. [6] Methods

We consider two fourth-order iterative methods presented in [6,7]. Since these methods are well established, we state the particular expressions used to calculate the approximate solution of the given nonlinear equations and thus compare the efficiency of the new fourth-order iterative methods.

The first of fourth-order method presented by Soleymani et al. [6], is expressed as

$$y_n = x_n - \left(\frac{2}{3}\right)\left(\frac{f(x_n) f'(x_n)}{f'(x_n)^2 - f(x_n) f''(x_n)}\right), \tag{31}$$

$$x_{n+1} = x_n - 4\left[\frac{1+\left(\frac{3}{4}\right)^2 (w_n - 1)^2}{(4 - u_n - 3v_n)}\right]\left(\frac{f(x_n)}{f'(x_n)}\right), \tag{32}$$

Where

$$u_n = \frac{f(x_n) f''(x_n)}{f'(x_n)^2}, \tag{33}$$

$$v_n = \frac{f(y_n) f''(y_n)}{f'(y_n)^2}, \tag{34}$$

$$w_n = \frac{1 - v_n}{1 - u_n}. \tag{35}$$

The second of fourth-order method is by Soleymani et al. [7], and is given as

$$y_n = x_n - \left(\frac{2}{3}\right)\left(\frac{f(x_n) f'(x_n)}{f'(x_n)^2 - f(x_n) f''(x_n)}\right), \tag{36}$$

$$x_{n+1} = y_n - \left[2^{-3}\left(17 - 16w_n + 7w_n^2\right)\left(\left(\frac{f(x_n)\left(2f'(x_n) + t_n f''(x_n)\right)}{f'(x_n)^2}\right) + t_n (v_n - 2)\right)\right]\left(2 - 2v_n\right)^{-1} \tag{37}$$

where $t_n = x_n - y_n$ and $u_n, v_n, w_n$ are given by (33), (34), (35) respectively.

# 5 Numerical Examples

We present some numerical results obtained by the new iterative methods (6), (26) to solve some nonlinear equations with unknown multiplicity *m*. We have demonstrated the performance of the new fourth order iterative methods by using ten particular nonlinear equations. The stability and consistency of the results are determined by examining the convergence of the new iterative methods. Consequently, the errors obtained by each of the methods are displayed in the following tables. Furthermore, the errors displayed are of absolute value and inappropriate approximations by the various methods have been omitted in the following tables. The numerical results listed in the tables were performed on an algebraic system called Maple.

The new fourth-order Schroder-type methods requires six function evaluations and has the order of convergence four. We use definition 3 to evaluate the efficiency index of the new methods. Therefore, the efficiency index of the new methods given by (6) and (26) is $\sqrt[6]{4}=\sqrt[3]{2}$, which is equivalent to the Soleymani et al. [6] methods (32) and (37). The efficiency index of the third-order method (25) $\sqrt[4]{3}\approx 1.316$, whereas the efficiency index of the second-order method (5) is given by $\sqrt[3]{2}\approx 1.260$. We observe that the efficiency index of the new fourth-order methods has not been improved, however we have constructed new simple and effective methods when compared to the established fourth order methods [6,7]. The test functions, multiplicity *m* and their exact root $\alpha$ are displayed in Table 1. The difference between the root $\alpha$ and the approximation $x_n$ for test functions with initial guess $x_0$ are displayed in Table 2. Table 2 shows the absolute errors obtained by each of iterative methods described, we find that the new fourth order method is producing better results than the established methods. In addition, the computational order of convergence (COC) are displayed in Table 3. From the Table 3, we observe that the COC perfectly coincides with the theoretical result. In addition, the approximation $\hat{m}$ based on the present methods (5) are displayed in Table 4. In Table 4 we display the approximations of the multiplicity *m* obtained by the Thukral third-order formula (5), we see a remarkable precision of the multiplicity *m*. Moreover, $x_n$ is calculated by using the same total number of function evaluations for all methods.

**Table 1. Test functions, multiplicity *m*, root $\alpha$ and initial guess $x_0$**

| Functions | *m* | Roots | Initial guess |
|---|---|---|---|
| $f_1(x)=\left(x^6-1\right)^m$ | $m=2$ | $\alpha=-1$ | $x_0=-1.2$ |
| $f_2(x)=\left(x^3+4x-10\right)^m$ | $m=3$ | $\alpha=1.365230...$ | $x_0=1.1$ |
| $f_3(x)=\left((x-1)^3-1\right)^m$ | $m=4$ | $\alpha=2$ | $x_0=2.2$ |
| $f_4(x)=\left(\exp\left(x^2+7x-30\right)-1\right)^m$ | $m=5$ | $\alpha=2.842438...$ | $x_0=3$ |
| $f_5(x)=\left(\cos(x)+x\right)^m$ | $m=6$ | $\alpha=-0.739085...$ | $x_0=-1$ |
| $f_6(x)=\left(\sin(x)^2-x^2+1\right)^m$ | $m=7$ | $\alpha=1.404491...$ | $x_0=1.7$ |
| $f_7(x)=\left(e^{-x^2}-e^{x^2}-x^8+10\right)^m$ | $m=8$ | $\alpha=1.239417...$ | $x_0=1.5$ |
| $f_8(x)=\left(6x^5+5x^4-4x^3+3x^2-2x+1\right)^m$ | $m=9$ | $\alpha=-1.57248...$ | $x_0=-1.8$ |
| $f_9(x)=\left(\tan(x)-e^x-1\right)^m$ | $m=10$ | $\alpha=1.371045...$ | $x_0=1.2$ |
| $f_{10}(x)=\left(\ln\left(x^2+3x+5\right)-2x+7\right)^m$ | $m=11$ | $\alpha=5.469012...$ | $x_0=5$ |

**Table 2. Comparison of iterative methods**

| $f_i$ | **(5)** | **(25)** | **(6)** | **(26)** | **(32)** | **(37)** |
|---|---|---|---|---|---|---|
| $f_1$ | 0.433e-5 | 0.410e-71 | 0.141e-79 | 0.777e-145 | 0.233e-37 | 0.153e-83 |
| $f_2$ | 0.396e-13 | 0.493e-81 | 0.815e-219 | 0.271e-220 | 0.118e-198 | 0.241e-241 |
| $f_3$ | 0.211e-11 | 0.393e-76 | 0.399e-93 | 0.460e-147 | 0.411e-41 | 0.106e-60 |
| $f_4$ | 0.302e-17 | 0.307e-106 | 0.367e-142 | 0.478e-220 | 0.214e-62 | 0.395e-71 |
| $f_5$ | 0.704e-21 | 0.129e-76 | 0.154e-173 | 0.335e-189 | 0.652e-79 | 0.322e-75 |

***Table 2 continued….***

| | | | | | | |
|---|---|---|---|---|---|---|
| $f_6$ | 0.245e-11 | 0.347e-56 | 0.232e-93 | 0.694e-115 | 0.925e-48 | 0.311e-56 |
| $f_7$ | 0.499e-3 | 0.773e-64 | 0.235e-23 | 0.304e-89 | 0.582e-2 | - |
| $f_8$ | 0.765e-6 | 0.927e-39 | 0.231e-46 | 0.417e-65 | 0.323e-16 | 0.343e-33 |
| $f_9$ | 0.310e-4 | 0.135e-26 | 0.176e-30 | 0.122e-47 | 0.269e-12 | 0.880e-15 |
| $f_{10}$ | 0.219e-34 | 0.284e-40 | 0.554e-291 | 0.157e-190 | 0.135e-123 | 0.432e-125 |

**Table 3. Performance of COC**

| $f_i$ | **(5)** | **(25)** | **(6)** | **(26)** | **(32)** | **(37)** |
|---|---|---|---|---|---|---|
| $f_1$ | 1.9601 | 3.0000 | 4.0000 | 4.0000 | 3.9988 | 4.0000 |
| $f_2$ | 2.0000 | 2.9888 | 4.0000 | 4.0000 | 4.0000 | 4.0000 |
| $f_3$ | 1.9998 | 2.9872 | 4.0000 | 4.0000 | 4.0000 | 4.0000 |
| $f_4$ | 2.0000 | 3.0000 | 4.0000 | 4.0000 | 4.0000 | 4.0000 |
| $f_5$ | 2.0000 | 2.9874 | 4.0000 | 4.0000 | 4.0000 | 4.0000 |
| $f_6$ | 1.9999 | 2.0768 | 4.0000 | 4.0000 | 4.0000 | 4.0000 |
| $f_7$ | 1.7889 | 3.0000 | 3.9997 | 4.0000 | 3.0373 | - |
| $f_8$ | 1.9840 | 3.0000 | 4.0000 | 4.0000 | 4.0000 | 4.0000 |
| $f_9$ | 1.8481 | 2.9994 | 4.0000 | 4.0000 | 3.9995 | 3.9999 |
| $f_{10}$ | 3.0000 | 3.0000 | 4.0000 | 4.0000 | 4.0000 | 4.0000 |

**Table 4. Performance of multiplicity *m*, based on the approximants of (25)**

| $f_i$ | **(5)** | **(25)** | **(6)** | **(26)** | **(32)** | **(37)** |
|---|---|---|---|---|---|---|
| $f_1$ | 2.0000 | 2.0000 | 2.0000 | 2.0000 | 2.0000 | 2.0000 |
| $f_2$ | 3.0000 | 3.0000 | 3.0000 | 3.0000 | 3.0000 | 3.0000 |
| $f_3$ | 4.0000 | 4.0000 | 4.0000 | 4.0000 | 4.0000 | 4.0000 |
| $f_4$ | 5.0000 | 5.0000 | 5.0000 | 5.0000 | 5.0000 | 5.0000 |
| $f_5$ | 6.0000 | 6.0000 | 6.0000 | 6.0000 | 6.0000 | 6.0000 |
| $f_6$ | 7.0000 | 7.0000 | 7.0000 | 7.0000 | 7.0000 | 7.0000 |
| $f_7$ | 7.9835 | 8.0000 | 8.0000 | 8.0000 | 7.9999 | - |
| $f_8$ | 9.0000 | 9.0000 | 9.0000 | 9.0000 | 9.0000 | 9.0000 |
| $f_9$ | 9.9967 | 10.000 | 10.000 | 10.000 | 10.000 | 10.000 |
| $f_{10}$ | 11.000 | 11.000 | 11.000 | 11.000 | 11.000 | 11.000 |

# 6 Conclusion

In this work, we have presented two new multi-point iterative methods for solving nonlinear equations with multiple roots. We have shown analytically and numerically, that the new Schroder-type iterative methods converge to the order four. The objective for presenting these new iterative methods were to establish a higher order of convergence method than the existing third-order methods [9]. Furthermore, we have found that the results obtained by the new Schroder-type methods are producing better approximation of the zeros than the other well established methods, namely the Soleymani et al. methods (32) and (37). Finally, the new Schroder-type methods are simple, very effective and robust.

## Competing Interests

Author has declared that no competing interests exist.

## References

[1] Gautschi W. Numerical analysis: An introduction. Birkhauser; 1997.

[2] Lagouanelle JL. Sur une mtode de calcul de l'ordre de multiplicity des zros d'un polynme. C. R. Acad. Sci. Paris Sr. A. 1966;262:626-627.

[3] Petkovic MS, Neta B, Petkovic LD, Dzunic J. Multipoint methods for solving nonlinear equations. Elsevier; 2012.

[4] Noor MA, Shah FA. A family of iterative schemes for finding zeros of nonlinear equations having unknown multiplicity. Appl. Math. Inf. Sci. 2014;8:2367-2373.

[5] Schroder E. Uber unendich viele Algorithmen zur Auflosung der Gleichungen. Math. Ann. 1870;2: 317-365.

[6] Soleymani F, Babajee DKR. Computing multiple zeros using a class of quartically convergent methods. Alex. Eng. 2013;52:531-541.

[7] Soleyani F, Shateyi S, Ozkum G. An iterative solver in the presence and absence of multiplicity for nonlinear equations. The Scientic World Journal; 2013. Article ID 837243.

[8] Thukral R. New variants of the Schroder method for finding zeros of nonlinear equations having unknown multiplicity. J. Adv. Math. 2014;8(3):1675-1683.

[9] Thukral R. New third-order Schroder-type method for solving nonlinear equations having unknown multiplicity. Amer. J. Comput. Appl. Math. 2015;5(5):147-153.

[10] Traub JF. Iterative methods for solution of equations. Chelsea Publishing Company, New York; 1977.



***Peer-review history:***
*The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)*
*http://sciencedomain.org/review-history/12331*